Intelligent Wind Turbine Power Curve Modelling Using the Third Version of Cultural Algorithm (CA3)

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Abstract- The wind turbine generator power curve (WTGPC) gives the relationship between the wind speed and power output of the wind turbine at any given time. The power curves, which are usually provided by the manufacturer company, are mainly used in forecasting, energy planning and performance monitoring of wind turbines. The WTGPC model plays a significant role in the control and monitoring of wind farms as well as playing a role in the wind farms power injection to the grid. This paper presents a comprehensive analysis of several methods of modelling the WTGPC, with respect to four commercial wind turbines; 330, 900, 2000 and 3050 kW. In the first step, the proposed method of the study, based on quadratic Gaussian function, is compared to several developed mathematical models by using error measurement techniques including the mean square error (MSE) and residual analysis. The accuracy of the proposed method has then been improved by means of the third version of the cultural algorithm (CA3) through the optimization of the proposed method coefficients. The ultimate performance of the compared methods has been investigated by the normalized root mean squared error (NRMSE), where the proposed method of the study shows an excellent performance for modelling of wind turbine power curves.

Keywords third version of the cultural algorithm (CA3), error analysis, mathematical modelling, quadratic Gaussian function, wind turbine generator power curve (WTGPC).

Nomenclature

Indexes

CA3	Third version of the Cultural Algorithm
FA	Firefly algorithm
GA	Genetic algorithm
GAPSO	Hybrid genetic algorithm and particle swarm optimisation
PSO	Particle swarm optimisation
MAE	Mean absolute error
MSE	Mean squared error
NRMSE	Normalised root mean square error

WTGPC	Wind turbine generator power curve

Variables

Λ	Minimum asymptote of a logistic function
A	Swept area of wind turbine generator
$a_1, \ldots, a_6$	Coefficients of the polynomial functions
B	Hill's slope of a logistic function
C	Inflection point of a logistic function
D	Maximum asymptote of a logistic function
ρ	Density of air
E	Asymmetry factor of a logistic function
$I_j(t)$	Closed interval of $N(t)$
l, u	The lower and upper bound which are initialized by the domain values
$L_j(t)$	Score of the lower bound at $N(t)$
$N(t)$	Normative knowledge component of the cultural algorithm
N_{ij}	A normalized number for individual i and component j
n	Number of data points
n_s	Number of variables of the situational component
n_x	Number of variables of the normative component
$P(v)$	Power output of wind turbine generator
P_{rate}	Power rate of wind turbine generator
v_i	Wind speed
v_{cin}	Cut-in speed of wind turbine generator
v_{cout}	Cut-out speed of wind turbine generator
v_{rate}	Rated speed of wind turbine generator
$S(t)$	Situational knowledge component of the cultural algorithm
δ_j	Step size of the belief interval
${\delta_j}^2(t)$	The variance of normalized number N_{ij}
$U_j(t)$	Score of the upper bound of $N(t)$
$X_j(t)$	Dimension of belief space at component j
$X_l(t)$	An accepted response
$x_{ij}(t)$	The mean of normalized number N_{ij}
$x_{lj}(t)$	An accepted response of the component j
$\acute{x}_{ij}(t)$	Influence function
$x_j^{min}(t)$	Minimum boundary of the closed interval at generation t
$x_j^{max}(t)$	Maximum boundary of the closed interval at generation t
x_i^{Actual}	Actual value (observed value)
$x_i^{estimated}$	Estimated value
$\hat{y}(t)$	Best individual of the solution vector

1. Introduction

Renewable energy is an important alternative source to produce electricity not only due to the limited sources of fossil fuels but also due to their negative effects on the environment. To mitigate the aforementioned difficulties and reduce reliance on fossil fuels, deployment of renewable energies such as the wind, solar and tide energies has grown rapidly in recent years. Among those renewable energies, wind energy has been given attention by scientists and governments worldwide and its global installation reached nearly 370 GW at the end of the year 2014 [1]. The nature of intermittency and volatility of wind energy, however, presents challenges when the penetration of wind power into the conventional power grids is high [2].

To overcome the issue of the integration of wind energy into power systems, having precise prediction models to estimate wind power are vital. This estimation can be obtained while converting the available wind speed into actual wind power by deploying a wind turbine performance power curve, from the manufacturer, and its nonlinear relationship with wind speed [3-4]. Generally, wind turbines are tested by the manufacturer under ideal weather conditions and therefore the provided power curves do not consider the power output of wind turbines in any abnormal weather condition. Some important factors such as the location of the turbine, the velocity and the direction of wind could cause considerable differences between experimental power curves and theoretical power curves [5].

Precise estimation of wind energy potential needs detailed information about the location where the wind turbine will be installed. Wind speed varies according to geographical location and its variation can be described by different probability distribution functions (PDFs). Some of these functions are Weibull, Rayleigh, Gamma, Beta, Lognormal and Logistical functions. Due to the uncertain behaviour of wind speed, one of the best approaches is to apply statistical methods to consider the intermittency of wind variations in converting it to electrical power [6-7]. There are several statistical methods to model a wind turbine power curve, and they are classified into parametric and nonparametric methods. Parametric methods, which are based on mathematical functions, include polynomial regression, segmented linear and logistic distribution (that is based on probabilistic distributions). Nonparametric methods include neural networks, fuzzy logic methods, and data mining methods. One of the advantages of nonparametric methods over parametric ones is that they do not require any pre-specified models and they can precisely model a wide range of possible shapes of the power curve.

In 2017, Ouyang et al. proposed a method based on the centres of data partitions and data mining [8]. To adopt the data partitions approach for modelling of the WTG power output, they have divided the wind power curve into unit intervals in which the values of each centre has been calculated. A support vector machine (SVM) technique was then used to reconstruct the wind power curve based on the captured field data. In 2016, Villanueva and Feijoo proposed a procedure based on the 4-parameter logistic function to model the wind the power curve parameters supplied by the manufacturer [9]. The proposed method has the capability to be converted to a 3-parameter logistic function in case of less complicated power curves, where a continuous function is employed to simplify the behaviour of the power curves in different classes of wind turbines.

In 2014, Shokrzadeh et al. investigated the wind turbine power curve models according to four parametric and non-parametric models [10]. The polynomial regression (PR) is considered as the benchmark in parametric modelling, and all the related problems with this method have been discussed. They have introduced a method according to locally weighted polynomial regression (LWPR) technique and compared its advantages over the PR. To elaborate their analysis a regression spline method was studied to obtain more flexibility in estimating any wind power curve.

In 2009, Kusiak et al. developed three new power curve models based on the least squares method, maximum likelihood estimation method, and a non-parametric model [11]. All the coefficients and parameters of the least squares method and maximum likelihood estimation method have been defined by means of an evolutionary strategy algorithm, where a k-nearest neighbour (k-NN) algorithm has been used to evaluate the coefficients of the non-parametric model. Due to the excellent performance of the least squares model and the non-parametric for all the studied cases, they have been recommended for the on-line monitoring of the power curves.

In 2015, Das and Mazumdar conducted a comprehensive study to investigate the performance of over 150 commercially available wind turbines according to their manufacturer data on wind speed variation and corresponding power outputs [12]. The study has considered a number of methods such as normalized expressions of the linear, cubic, quadratic, n-order and a two-parameter approach for modelling of wind power output. In order to assess the performance of the mentioned models, the two widely used methods for the goodness of fit evaluation (root mean square error (RMSE) and r-squared) have been used.

In 2016, Xu et al. developed a sophisticated local polynomial regression (LPR) algorithm to obtain an adaptive model of the time independent wind power curves for predicting applications [13]. They proposed a new approach based on the data-driven bandwidth selection method. The proposed method has been formulated through a combination of block-wise least-squares parabolic fitting and the probability integral transform.

In 2015, Yip et al. presented a model to estimate the capacity factor of the wind turbines through a mathematical model according to curve fitting regression [14]. Thereafter, to verify the accuracy of the calculated energy output of the wind turbines, the obtained results have been compared with a commercial software namely WASP. They have used four different classes of Vestas's wind turbines to validate the practicality of their methods.

In 2013, Zarate-Minano et al. developed two approaches for implementing wind speed models based on stochastic differential equations (SDE) [15]. The developed models are expected to produce wind speed trajectories with the same statistical properties of the captured historical wind speed data which is available for a specific location. The developed models were very robust and simplified in such a way that they only used the observed autocorrelation information and marginal distribution of the wind speed data. As the developed models have used a continuous function, they had the ability to reconstruct the wind speed trajectories at any given time. This ability was however constrained to time frames as diurnal and seasonal properties were not taken into account.

In this paper to the model power output of the selected WTG a new nonparametric method, which was based on quadratic Gaussian function, has been presented and compared with several developed mathematical models. To evaluate the performance and accuracy of the proposed method over the other studied methods in this literature, mean square error and residual analysis have been deployed. To improve the proposed method, the third version of the cultural algorithm (CA3) has been applied to obtain the optimal coefficients of the introduced method and the normalized root mean squared error (NRMSE) has been employed to find out the functionality of the proposed method and its accuracy over different methods.

The paper organized as follows. Section 2 includes mathematical modelling of WTG power curve and the goodness of fit evaluation. The conceptual and mathematical formulation of CA3 has been introduced in section 3. Section 4 comprises of the results and discussions and is followed by the conclusion in section 5.

2. Problem Formulation

2.1. *Mathematical Modelling of Wind Turbine Generator Power Curves*

The actual power output of wind turbine power curves is calculated based on the following model [16-17]:

$$P(v) = \begin{cases} 0 & : v_i < v_{Cin}\ , v_i > v_{Cout} \\ P(v) & : \quad v_{Cin} \le v_i \le v_{rate} \\ P_{rate} & : v_{rate} < v_i < v_{Cout} \end{cases} \tag{1}$$

The above equation is used to solve and estimate the power output of wind turbines according to its manufacturer characteristics such that the calculation error is minimized. Generally, the *P(v)* can be modelled using several methods such as operational characteristics of wind turbine parameters, manufacturer power curve output modelling and physical characteristics of WTG. The first set of these techniques, which are based on the operational characteristics wind turbine parameters, can be formulated through the following models:

2.1.1 Model No.1 (M_1): This model calculates the power output of WTG through a linear equation:

$$M_1 = P_{rate} * \left(\frac{v_i - v_{cin}}{v_{rate} - v_{cin}}\right) \tag{2}$$

2.1.2 Model No.2 (M_2): This model represents the M1 in a quadratic form, where each of the wind turbine parameters have been individually squared.

$$M_2 = P_{rate} * \left(\frac{v_i^2 - v_{cin}^2}{v_{rate}^2 - v_{cin}^2}\right) \tag{3}$$

2.1.3 Model No.3 (M_3): This model is another modified model of M1 where the fraction part of the equation has been cubed.

$$M_3 = P_{rate} * \left(\frac{v_i - v_{cin}}{v_{rate} - v_{cin}}\right)^3 \tag{4}$$

2.1.4 Model No. 4 (M_4): This model can be assessed based on the ratio of the input wind speed at any given time over the rated wind speed of WTG.

$$M_4 = P_{rate} * \left(\frac{v_i^3}{v_{rate}^3}\right) \tag{5}$$

The second sets of the developed models are based on the mathematical relationship of the wind speed at different levels and the actual power output of WTG which has been captured by the manufacturer in the laboratory. These models are referred to as non-parametric models, as the characteristics of WTG do not have any effect on the determination of the power output of WTG. The coefficients of these models are derived according to least-square approximation at different degrees of polynomials [18]:

2.1.5 Model No. 5 (M_5): This model estimates the power output of WTG through the second degree of the polynomial:

$$M_5 = (a_1 v_i^2 + a_2 v_i + a_3) \tag{6}$$

subject to

$$\begin{bmatrix} \sum_i^n x_i^4 & \sum_i^n x_i^3 & \sum_i^n x_i^2 \\ \sum_i^n x_i^3 & \sum_i^n x_i^2 & \sum_i^n x_i \\ \sum_i^n x_i^2 & \sum_i^n x_i & n \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum_i^n x_i^2 y_i \\ \sum_i^n x_i y_i \\ \sum_i^n y_i \end{bmatrix} \tag{7}$$

Equation 6 demonstrates that the coefficients of M5 (which is a quadratic equation) can be derived from the least-square approximation method. It is significant to note that the coefficients of M5 can also be evaluated by means of a binomial process, where the power output of the binomial function is required to be multiplied by the power rate of the WTG. The binomial calculation of the coefficients can be illustrated based on the wind turbine parameters:

$$a_1 = \frac{1}{(v_i - v_{rate})^2}\left[2 - 4\frac{(v_i + v_{rate})^3}{2v_{rate}}\right] \tag{8}$$

$$a_2 = \frac{1}{(v_i - v_{rate})^2}\left[4(v_i + v_{rate})\frac{(v_i - v_{rate})^3}{2v_{rate}} - 3(v_i + v_{rate})\right] \tag{9}$$

$$a_3 = \frac{1}{(v_i - v_{rate})^2}\left[v_i(v_i + v_{rate}) - 4v_i v_{rate}\frac{(v_i - v_{rate})^2}{2v_{rate}}\right] \tag{10}$$

Higher degrees of the polynomial can be achieved with the same procedure as explained in eq. 6, where the dimensions of the right and left-hand side of the matrix would be increased with respect to the degree of the desired polynomial equation. In order to have a precise investigation of the accuracy of the power output of WTG by increasing the degree of the polynomial equations, the study has considered four more models based on the third, fourth, fifth and sixth degree of polynomial regression. The considered models can be expressed as follows [16]:

2.1.6 Model No. 6 (M_6):

$$M_6 = (a_1 v_i^3 + a_2 v_i^2 + a_3 v_i + a_4) \tag{11}$$

2.1.7 Model No. 7 (M_7):

$$M_7 = (a_1 v_i^4 + a_2 v_i^3 + a_3 v_i^2 + a_4 v_i + a_5) \tag{12}$$

2.1.8 Model No. 8 (M_8):

$$M_8 = (a_1 v_i^5 + a_2 v_i^4 + a_3 v_i^3 + a_4 v_i^2 + a_5 v_i + a_6) \tag{13}$$

2.1.9 Model No. 9 (M_9):

$$M_9 = (a_1 v_i^6 + a_2 v_i^5 + a_3 v_i^4 + a_4 v_i^3 + a_5 v_i^2 + a_6 v_i + a_7) \tag{14}$$

Models M6, M7, M8 and M9 are based on the third, fourth, fifth and sixth degrees of polynomial regression respectively.

The third set of the developed models are based on the logistic parameter functions, which can be considered as a sub-category of manufacturer power curve output modelling. The logistic parameter functions are formulated according to nonlinear regression modelling which is commonly used for curve-fitting analysis in bioassays, immunoassays or dose-response curves. In general, the logistic parameter functions can be divided into three categories; three logistic parameter regression (LPR3), four logistic parameter regression (LPR4) and five logistic parameter regression (LPR5). Like the non-parametric models, LPR3, LPR4 and LPR5 use the exponential form of least squares regression to evaluate the function coefficients. LPR3 is characterized by sigmoidal shape function that fits the top plateaus of the curve and the slope factor (Hill's slope). This curve is symmetrical around its inflection point [19]. Equation (14) represents the mathematical formulation of LPR3:

2.1.10 Model No. 10 (M_{10}):

$$M_{10} = \frac{D}{\left(1 + \left({}^{v_i}/_{C}\right)^{-B}\right)} \tag{15}$$

The LPR4 was developed to improve the performance of LPR3 by considering the minimum asymptotes of the given data.

2.1.11 Model No. 11 (M_{11}):

$$M_{11} = D + \frac{(\Lambda - D)}{\left(1 + \left({}^{v_i}/_{C}\right)^{B}\right)} \tag{16}$$

The LPR5 equation is equivalent to the LPR4 equation with an additional parameter added for asymmetry [20]. This additional parameter provides a better fit for LPR5 in comparison to the other models of logistic parameter functions.

2.1.12 Model No. 12 (M_{12}):

$$M_{12} = D + \frac{(\Lambda - D)}{\left(\left(1 + \left({}^{v_i}/_{C}\right)^{B}\right)\right)^{E}} \tag{17}$$

The study has proposed two new methods for modelling the WTG power curve based on a quadratic Gaussian function and a sine function. The detailed procedure for evaluating the coefficients of the model can be found in [21]. The proposed methods can be formulated as follows:

2.1.13 Model No. 13 (M_{13}): this model is based on a quadratic Gaussian function, where the formulation can be expressed as:

$$M_{13} = a_1 e^{\left[-\left(\frac{v_i - b_1}{c_1}\right)^2\right]} + a_2 e^{\left[-\left(\frac{v_i - b_2}{c_2}\right)^2\right]} \tag{18}$$

where

a = Amplitude of the curve.
b = Centroid (location) of the curve.
c = Peak width of the curve.

2.1.14 Model No. 14 (M_{14}): this model is based on the sine function, where the formulation can be represented as:

$$M_{14} = a_1 \sin(b_1 v_i + c_1) + a_2 \sin(b_2 v_i + c_2) \tag{19}$$

where
a = Amplitude of the curve.
b = Frequency of the curve.
c = Phase constant for each sine wave term.

One of the most significant features of both of the proposed methods is that it considers every nonlinear behaviour or erratic movement of the given curve and this enables the method to handle the nonlinearity for the curve fitting purposes.

The fourth set of the developed models are based on physical and mechanical characteristics of the WTG. These models are derived through to the fundamental theorem of the Betz model which simulates and converts the mechanical aspects of WTG into measurable power output. Losses in efficiency for a practical WTG are caused by the viscous and pressure drag on the rotor blades, the swirl imparted to the air flow by the rotor, and the power losses in the transmission and electrical system. In addition, uniformity is assumed over the entire swept area of the rotor, and the speed of the air beyond the rotor is considered to be axial. The ideal wind rotor is taken at rest and is placed in a moving fluid atmosphere [22]. The derivation of the Betz model can be expressed as follows:

The initial step is from the kinetic energy:

$$E = \frac{1}{2} m v^2 \tag{20}$$

The power that can be harnessed from the wind is given by the rate of change of energy which is equal to the rate of change in mass:

$$P = {}^{dE}/_{dt} = \frac{1}{2} v^2 \, {}^{dm}/_{dt} \tag{21}$$

subject to

$$\begin{cases} {}^{dm}/_{dt} = \rho A \, {}^{dx}/_{dt} \\ {}^{dx}/_{dt} = \quad v \end{cases} \tag{22}$$

By equalizing the abovementioned relationships we have

$${}^{dm}/_{dt} = \rho A v \tag{23}$$

$$P = \frac{1}{2} \rho A v^3 \tag{24}$$

Eq. 23 represents the basic form of the Betz method for the modelling of the WTG power output. This study has considered three scenarios for the Betz model through to its capacity ratios and wind speed exponents to present a more comprehensive investigation of the Betz concept. The considered scenarios of the Betz model can be described in the following models [16]:

2.1.15 Model No. 15 (M_{15}):

$$M_{15} = \frac{1}{2} \rho A C_{p,eq} v_i^3 \tag{25}$$

In M15 which is the cubic power curve of Betz model, a capacity equal ratio ($C_{p,eq}$) has been considered where the ratio is equal to 0.45 and the air density (ρ) is equal to 1.225.

2.1.16 Model No. 16 (M_{16}):

$$M_{16} = \frac{1}{2} \rho A C_{p,max} v_i^3 \tag{26}$$

In this model, which is based on the approximated cubic power curve, the maximum capacity ($C_{p,max}$) ratio is equal to 0.5.

2.1.17 Model No. 17 (M_{17}):

$$M_{17} = \frac{1}{2} \rho A K_p (v_i^{\beta} - v_{cin}^{\beta}) \tag{27}$$

The constant ratio of kinetic energy (K_p) is equal to 0.889 and wind speed exponent (β) is equal to 3. This model is called the exponential power curve of the Betz model.

2.2. The goodness of fit evaluation

In statistical analysis and studies, the performance of a mathematical method or distribution model to fit a given curve is assessed through the goodness of fit analysis. The goodness of fit analysis provides detailed information about the capability of a considered method to predict the observed or actual data.

In this study, to have a very precise and comprehensive investigation of proposed methods, four statistical methods have been used to evaluate their performances. Mean squared error (MSE) has been used to measure the difference between the observed data and the estimated values. In this study, it has been used as the objective function to reduce the errors of estimated values during the optimization process for evaluating the optimal coefficients of the proposed model. The mean absolute error (MAE) has been used as the second method to provide an overall comparison for all methods. The third method which has been used to measure the performance of the proposed method is the residual analysis. The residual analysis tells us about the location of estimated values in comparison to actual data. The fourth and the last method is the normalized root mean squared error (NRMSE). The models that have values closer to zero represent a better performance [23]. The formulation of all three methods can be expressed as follows:

$$Residual = x_{\mathrm{i}}^{Actual} - x_i^{estimated} \tag{28}$$

$$MSE = \frac{1}{n} \sum_{i=1}^{n} \left(x_i^{Actual} - x_i^{estimated} \right)^2 \tag{29}$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(x_i^{Actual} - x_i^{estimated} \right)^2} \tag{30}$$

$$NRMSE = \frac{RMSE}{x_i^{max} - x_i^{min}} \tag{31}$$

3. The Third Version of Cultural Algorithm (CA3)

Cultural Ecology refers to the study of the interaction of culture in the environment where cultural information can be transmitted to the future generations through an inheritance mechanism. In 1994, Reynolds adapted this concept to develop Cultural Algorithm [24] for use as an optimization technique. The objective of the algorithm is to process information at population and cultural (i.e. knowledge) level to improve the convergence of an evolutionary algorithm. At the population level, individuals represent candidate solutions, while at the cultural level, information acquired by individuals is stored and shared with the current generation [25].

The process of the selection of the individuals is based on many factors such as knowledge, physical appearance, and wealth. The generalization will take place from these individuals. Next is the adoption of the distilled knowledge by the population. The knowledge obtained by the individuals will be passed to the next generation to direct the behaviour of the population agents. As a result, we can say that CA implementation is based on two main components which are population space and the belief space. The population space is responsible for storing the information that is generated by the individuals, and the belief space maintains and distils this information during the process of evolution.

The general framework of CA is depicted in Figure 1. The general framework of CA is based on two spaces; population space and belief space. The population space contains all the possible individuals the can be considered for the optimization process. The belief space comprises a set of experience and knowledge structure of the individuals. The main tasks of the population space are to select, fit and variate the individuals of the population space, where the belief space is dedicated to adjusting of the accepted individuals. CA can be categorized into different versions based on their influence functions. The responsibility of the influence function is to affect the population according to the regulation of beliefs to determine the mutational step size and the direction of changes.

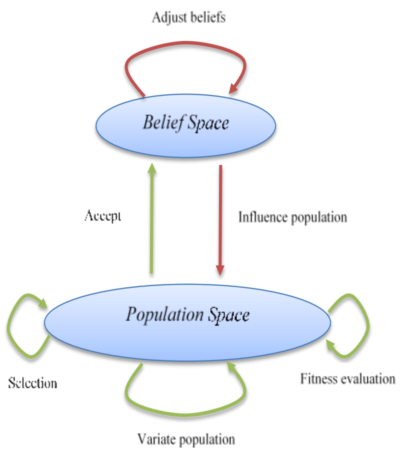

Fig. 1. General framework of cultural algorithm

Goudarzi, et al proposed four versions for CA, where the third version (CA3) was found as the most efficient version for the optimization purposes [26]. CA3 is based on two knowledge components; the situational knowledge and the normative knowledge. The situational knowledge component is in charge of finding the best possible solution in a generation, formulated as:

$$S(t+1) = \{\hat{y}(t+1)\} \tag{32}$$

where

$$\hat{y}(t+1) = \begin{cases} min_l\{X_l(t)\} & if\ f\left(min_{l=1,\dots,n_B(t)}\{X_l(t)\}\right) < f(\hat{y}(t)) \\ \hat{y}(t) & otherwise \end{cases} \tag{33}$$

subject to

$$X_l(t), l = 1, 2, 3, \dots, n_B(t) \tag{34}$$

$$n_B(t) = \left[\frac{n_{pop}\gamma}{t}\right], \quad \gamma \in [0,1] \tag{35}$$

The normative knowledge is the component which prepares different scales for each individual behaviour in order to direct them towards their mutational adjustments. The normative knowledge can be mathematically expressed as:

$$x_j^{min}(t+1) = \begin{cases} x_{lj}(t) & if\, x_{lj}(t) \le x_j^{min}(t)\ \ or\ \ f(X_l(t)) < L_j(t) \\ x_j^{min}(t) & otherwise \end{cases} \tag{36}$$

For updating $L_j(t)$

$$L_j(t+1) = \begin{cases} f(X_l(t)) & if\, x_{lj}(t) \le x_j^{min}(t)\ \ or\ \ f(X_l(t)) < L_j(t) \\ L_j(t) & otherwise \end{cases} \tag{37}$$

$$x_j^{max}(t+1) = \begin{cases} x_{lj}(t) & if\, x_{lj}(t) \ge x_j^{max}(t)\ \ or\ \ f(X_l(t)) < U_j(t) \\ x_j^{max}(t) & otherwise \end{cases} \tag{38}$$

For updating $U_j(t)$

$$U_j(t+1) = \begin{cases} f(X_l(t)) & if\, x_{lj}(t) \ge x_j^{max}(t)\ \ or\ \ f(X_l(t)) < U_j(t) \\ U_j(t) & otherwise \end{cases} \tag{39}$$

In this version, the step size would be defined by means of situational knowledge where the changes in direction would be carried out by normative knowledge. CA3 can be described as:

$$\acute{x}_{ij}(t) = \begin{cases} x_{ij}(t) + \left|\sigma_{ij}(t)N_{ij}(0,1)\right| & if\ x_{ij}(t) < \hat{y}_j(t) \\ x_{ij}(t) - \left|\sigma_{ij}(t)N_{ij}(0,1)\right| & if\ x_{ij}(t) > \hat{y}_j(t) \\ x_{ij}(t) + \sigma_{ij}(t)N_{ij}(0,1) & otherwise \end{cases} \tag{40}$$

subject to

$$\sigma_{ij}(t) = \alpha\left[x_j^{max}(t) - x_j^{min}(t)\right], \quad 0 < \alpha < 1 \tag{41}$$

4. Results and Discussion

In this study, the proposed method was used to model the output of the WTG power curve and included an improvement of the efficiency of the proposed method by the use of CA3 to optimize its coefficients. To examine the effectiveness of the proposed method, it has been tested on four different class of wind turbines. All comparison cases were performed to validate the suitability of the methodology of the study. All the mathematical formulation and algorithms have been implemented on MATLAB 2015a. They have been executed on a personal computer with the following specifications, Intel® Core™ i5-3210M (3.1 GHz), 6.00 GB RAM (DDR3) and win 8.1 operating system (OS). The study performed 20 trials for each class of WTG to produce reasonable results and the consideration of any associated error in calculations, while the maximum number of iterations for all the trials were fixed at 300. In this study, the adjusting parameters of the CA were as follows:

- Population size: 50
- Acceptance ratio: 0.15
- Strategy parameter: 0.25
- Scaling coefficient: 0.5

To demonstrate the practicality of the proposed method in the modelling of the WTG power curve, four statistical methods have been used to investigate the performance of the proposed method in comparison to the others. This section can be divided into two parts; the first part has attempted to show the superiority of the proposed method in comparison with other previously developed methods and the second part the performance of the proposed method has been improved by using CA3 to determine the coefficients of the proposed method in an optimal manner.

Figure 2 shows the residuals representation of the three best models for all the classes of WTGs. The residual values have been depicted against the actual measured values by the manufacturer. To have an accurate evaluation of residual behaviours, the confidence bound has been set as 33% of the highest deviation from the actual values in all comparison cases. The confidence bound has been depicted in the red dashed line. As seen from Figure 2, M13 has obtained fewer outliers in comparison to all other classes of WTGs aside from class E-33 where M14 showed a better performance. The placement of the residuals of M13 for the different classes of WTGs are as follows; 50% for the E-33, 76.92% for the E-44, 81.82% for the E-82 and 90.91% for the E-101.

Table 1 represents the comparison of MAE values for all the models, where the M13 has obtained the lowest values of MAE for all classes of WTGs aside from class E-33, where the M14 has achieved a slightly lower error. The MAE values of M13 are 2.265, 3.455, 11.752 and 27.767 for the E-33, E-44, E-82, and E-101, respectively. Figure 3 illustrates an overall comparison of NRMSE for all classes of WTGs for the different models. The colour-bar on the right-hand side of Figure 3 represents the details of measured values. It is observable that the proposed model of the study (M13) has an outstanding performance in comparison to the other compared models. From all the aforementioned results, it can be inferred that M13 has a good performance for modelling of WTG power curve in comparison to all previously developed models. In the second part of the result and discussion, the CA3 method has been used to find the optimal coefficients for the proposed model in order to enhance the accuracy of the proposed model for modelling of the WTG power curves. To demonstrate the capability of the proposed model, the obtained results of the proposed model have been compared to the other evolutionary algorithms.

Table 1. The MAE results for the all classes of WTGs

	MAE			
	E-33	E-44	E-82	E-101
M1	24.260	80.000	220.942	357.207
M2	36.688	95.040	239.110	385.025
M3	24.244	119.875	143.174	230.926
M4	46.618	191.628	257.757	400.765
M5	71.413	239.911	349.355	536.744
M6	33.127	58.034	207.966	335.616
M7	14.555	29.437	96.696	171.786
M8	15.083	23.904	106.361	183.986
M9	7.891	10.819	56.219	104.127
M10	9.495	20.286	56.777	117.359
M11	7.721	17.507	107.927	106.849
M12	1.763	4.245	18.501	53.808
M13	2.265	3.455	11.752	27.767
M14	1.580	6.177	18.780	38.886
M15	18.419	140.970	96.095	115.328
M16	23.941	179.114	121.640	189.973
M17	138.644	635.781	794.822	1211.272

Figure 4 to Figure 7 represents the optimization process of the proposed model coefficients in order to minimize the MSE for all the studied classes of the WTGs. As it can be seen from Figure 4, CA3, GAPSO and PSO have a very close behaviour during the optimization process for the WTG E-33 while the lowest MSE has been obtained by CA3 at 0.5466. The detailed comparison of the obtained MSE results for all the optimization algorithms with respect to the different classes of WTGs can be found in Table 2. From Figure 5 it is observable, for E-44 class of WTG, up until to 100th iteration GA has achieved a better performance during the optimization process but the best final value of MSE has been obtained by CA3 at 14.0924 followed by GAPSO and PSO at 14.3435 and 14.7978, respectively. Figure 6 represents the optimization process for E-82 WTG class, where CA3 shows a performance when compared to the other optimization algorithms. For this class of WTG, the best result has been obtained by CA3 at 36.0583. Figure 7 shows the comparison of optimization algorithms for E-101 WTG class. In this class, CA3 achieved the lowest value of MSE among the other algorithms at 63.2763, and this shows the capability of the proposed method. The final values of the optimal coefficients of the proposed model for different optimization algorithms are given in Table 3.

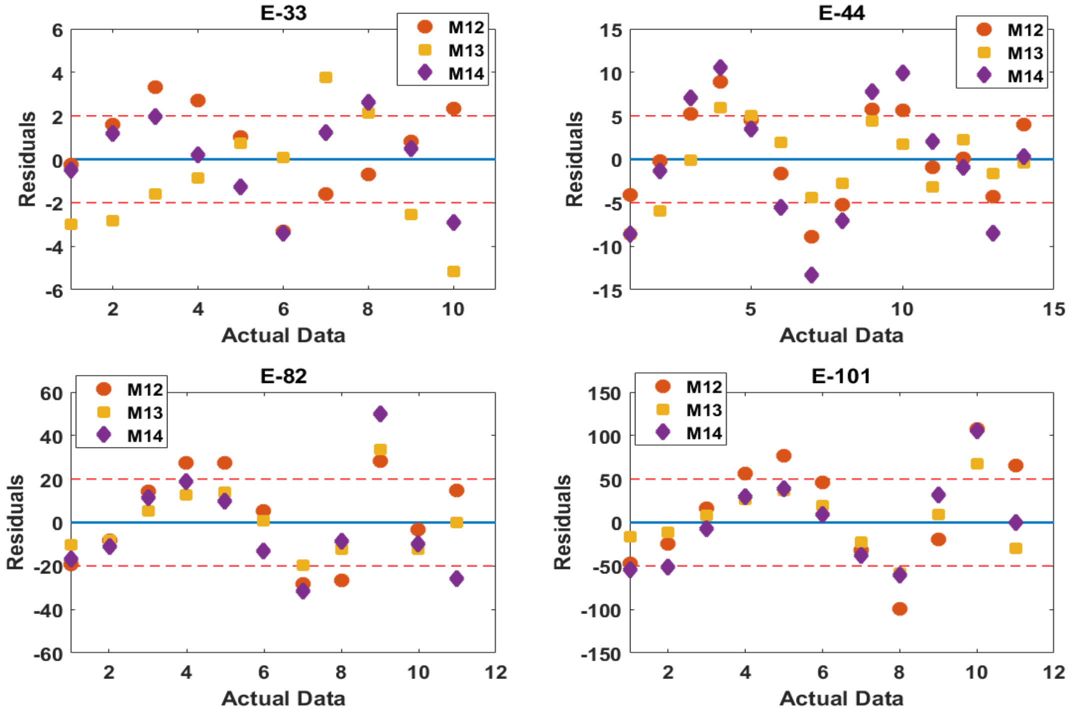

Fig. 2. The residuals representation of the three best models

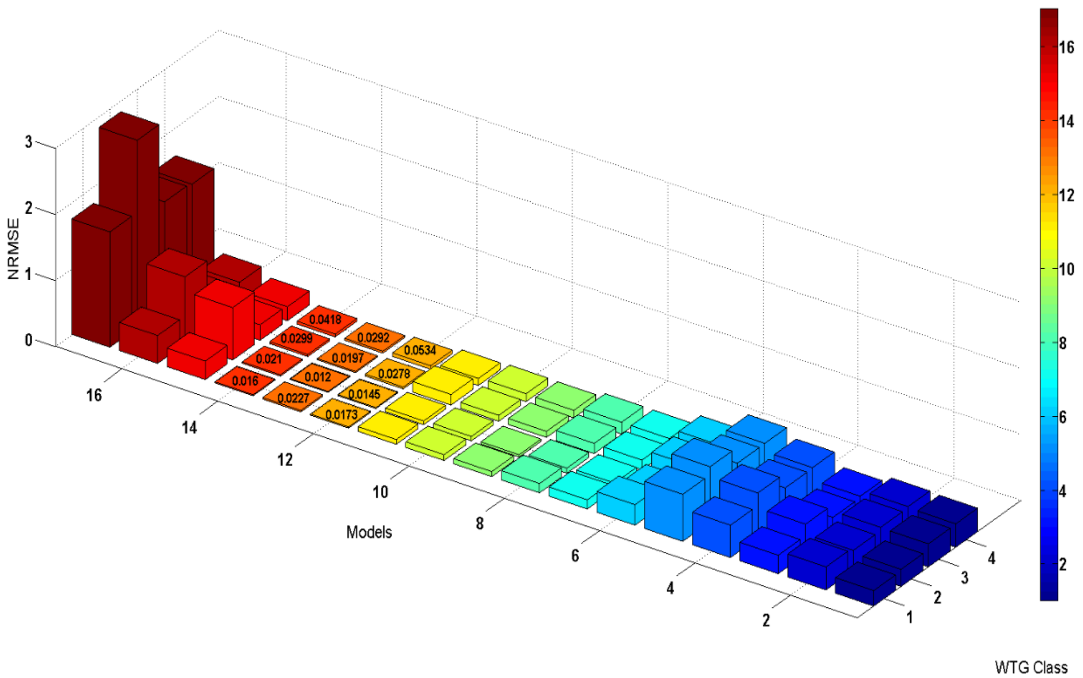

Fig. 3. Evaluation of NRMSE for all the studied classes of WTGs

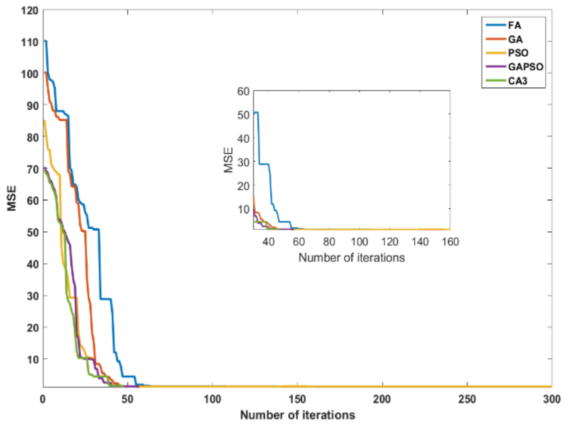

Fig. 4. The optimization process of the objective function (E-33 WTG)

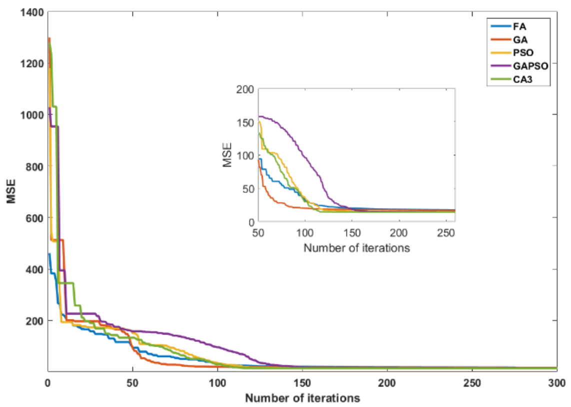

Fig. 5. The optimization process of the objective function (E-44 WTG)

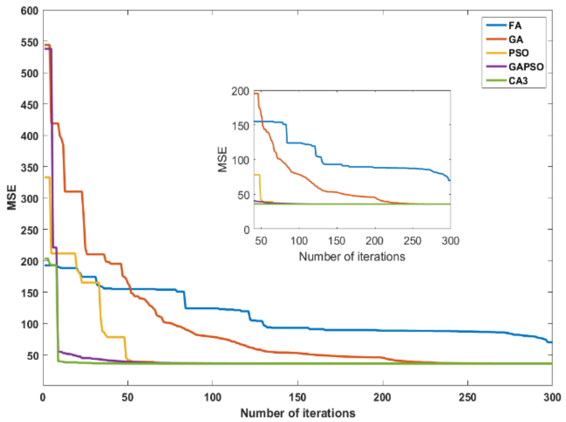

Fig. 6. The optimization process of the objective function (E-82 WTG)

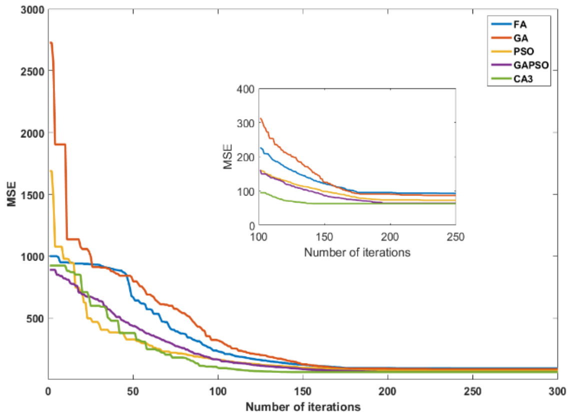

Fig. 7. The optimization process of the objective function (E-101 WTG)

Table 2. MSE results for all the optimization algorithms

WTG Class	FA	GA	PSO	GAPSO	CA3
E33	1.1734	1.23	1.2288	0.5897	0.5466
E44	15.7577	15.7286	14.4978	14.3435	14.0924
E82	70.1446	36.1032	36.0873	36.0663	36.0583
E101	92.9399	87.251	72.886	64.6689	63.2763

Table 3. The optimal coefficients of the proposed model for different optimization algorithms

Method		a1	b1	c1	a2	b2	c2
E33	FA	-2.79	2.03	5.19	322.69	12.49	4.94
	GA	3.18	-2.28	1.47	321.26	12.42	4.82
	PSO	-5.90	23.92	2.86	321.26	12.41	4.82
	GAPSO	-3.92	-1.95	9.37	323.42	12.50	4.94
	CA3	-3.28	7.86	183.98	326.41	12.50	4.96
E44	FA	199.72	16.84	-1.81	829.63	14.17	5.51
	GA	203.45	16.83	-1.82	828.31	14.16	5.50
	PSO	265.51	17.39	-2.24	819.75	14.08	5.45
	GAPSO	1091.97	21.39	-4.29	792.84	13.90	5.37
	CA3	1003.15	21.31	-4.55	774.48	13.77	-5.30
E82	FA	1988.92	12.41	4.70	66.54	10.07	0.41
	GA	2005.51	12.55	4.80	76.33	9.76	0.61
	PSO	2005.35	12.54	4.80	78.17	9.74	-0.59
	GAPSO	75.44	9.76	0.61	2005.37	12.54	4.81
	CA3	74.23	9.78	0.63	2005.59	12.55	4.81
E101	FA	3043.27	11.99	3.77	258.54	6.63	2.36
	GA	3042.34	11.98	-3.76	262.49	6.66	-2.33
	PSO	3043.51	11.99	3.73	281.19	6.73	2.45
	GAPSO	3038.12	11.96	3.71	284.92	6.74	-2.38
	CA3	3051.05	11.90	4.36	-253.17	9.00	-1.73

5. Conclusion

The precise modelling of WTG power curves is an essential tool for the wind farm operators and power system planners which can be used for evaluation and monitoring of the wind turbine's performance. The accurate modelling of WTG power output enables the system operator to consider the sufficient amount of reserve to handle the intermittency of the wind energy and enhance the penetration level of wind energy into the power grid which leads to the generation of power at the least cost. In this study, a non-parametric model, based on the quadratic Gaussian function, has been proposed for estimating WTG power curves. The proposed model has been benchmarked with several parametric and non-parametric models which have previously been developed for the modelling of WTG power curve. According to the results, it can be inferred that the parametric and polynomial regression models are constrained subject to their model characteristics and sensitivity of the error estimation. A precise fit to experimental data requires a higher degree of polynomial regression that sometimes can lead to over/under-fitting to the field data. In this regard, the study has used the cultural algorithm to find the optimal coefficients of the proposed model in order to reduce the estimating errors of the curve fitting and enhance the practicality of the model. The accuracy of the proposed model has been evaluated by four statistical methods, namely; residual analysis, MSE, MAE and NRMSE. The results of the study show that the proposed model presents a noticeably better performance in comparison to the other studied methods. In addition with the coupling of the proposed model to the cultural algorithm, the performance of the proposed model can be considerably improved. It is recommended that the proposed model of the study can be used in the power industry for the modelling and monitoring of the power outputs of wind turbine generators.

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