

An Analysis of Braking Energy Regeneration in Electric Vehicles

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Abstract- Proper utilization of waste energy can mitigate energy crisis and keep the cost under control. This project aimed to regenerate the energy from braking load in the vehicle. In this study, a prototype electric regenerative system is designed and tested experimentally to predict its performance. The prototype system is operated by a 12V motor that can recharge a 2.4V battery. During the experimental test, variable load is applied to test the time required for storing 0.84W into the battery. The results show that the battery is recharged by 0.80C at the time of rotation of 8sec applying the maximum load of 648g and recharged by 0.15C at the time of rotation 1.5sec applying the minimum load of 72g. It is observed that the amount of energy stored in the battery is increased with the increasing braking load. This energy can be used for operating auxiliary components such as fans, lights etc. and in cases, added to the main power source. In a nutshell, this project has brought the totally unused energy during braking in the limelight and paved the way to utilize this energy in a fruitful manner.

Keywords Electric vehicles (EVs), Brake load, Regenerative system.

Nomenclature

F_t	Transmitting load	V_m	Pitch line velocity
F_d	Dynamic load	T_p or N_p	No. of teeth of pinion
K_f	Strength reduction factor	T_g or N_g	No. of teeth of gear
P_d	Diametral pitch	D_p	Pitch diameter
S_a	Alternating component of total stress	E	Energy (Whr)
S_{as}	Variable stress in shear	i	Current (Ampere)
S_e	Equivalent stress	I	Moment of inertia
S_{es}	Static shear stress	J	Polar moment of inertia

S_m	Mean stress	N	Size factor
S_{ms}	Mean stress in shear	P	Power (W)
S_n	Fatigue strength	Q	Charge (coulomb)
S_{ns}	Shearing endurance strength	R	Resistance (Ω)
S_u	Ultimate strength	t	Time (sec)
S_y	Yield strength	T	Torque (Nm)
S_{ys}	Yield strength in shear	V	Voltage (volt)
		Y	Lewis form factor

1. Introduction

Energy is the fundamental needs in the modern era. Energy is directly linked to the well-being and prosperity across the world. Nowadays, the world is totally dependent on an abundant and uninterrupted supply of energy. As the sources of the conventional energy are not long lasting, so almost every sector including the transport sector is concentrated on the renewable sources of energy such as solar, wind, geothermal, tidal, biomass etc. [1]. Even the energy conversion from the waste materials such as pyrolysis is a good weapon to fight against the growing energy demand [2, 3]. Also, it cannot be denied that during the energy conversion a handsome amount of energy is wasted in vain. This wasted energy can be a good source of energy generation, which will be helpful to recover the energy demand.

In the transport sector, lots of energy is wasted without any recycling. Energy loss due to braking is one of them. Controlling an automobile vehicle's motion requires a brake. In the case of most conventional vehicles, the kinetic energy changed to the heat energy due to the friction between the pads and wheels. There is no possible way to store that heat energy and as results heat energy just flows away along with the air stream. A number of loss changes with how often, how hard and how long the brake is applied. In a word, it can be said that each time when the brake is applied to slow down or to stop the vehicles it causes the wastage of energy. To get the best use of this losses energy a Regenerative Braking System (RBS) is introduced [4-6]. If it is possible to successfully regenerate the energy large extents, then it will be a great achievement for the developing countries like Bangladesh.

So many researchers have studied the feasibility and practical implementation of the regenerative braking system in the field of Electrical Vehicles (EVs) [7-11]. However, some studies have been done under the urban driving conditions as well [12, 13]. Further studied have been carried on to develop the RBS models [10], simulation of the models [14] and controlling the motor [15]. Since the introduction of such system helps to reduce fuel cost, fuel consumption and gaseous emission [13, 16, 17], EVs also includes some problems or limitations such as security of the effective usage of battery and motor, long time battery charging and

short distance driving [18-20]. Moreover, the utilization of the waste energy can also play an important role in the recovery of regenerative braking energy and in the power quality improvement [21-23] and

The main concern of this study is to generate excess energy from the energy wasted for braking load and to achieve high efficiency and low cost by reducing the waste of energy [23]. A prototype of an electrical vehicle is designed and constructed for experimental analysis. In this study, it is confirmed that energy generates in every time for every load of breaking the system. This type energy regeneration is becoming one of the efficient and environmentally compatible technologies now a day. Brake energy regeneration helps to get the most out of the energy that is necessarily used for driving. In Bangladesh, the electric vehicles are on the rise like other countries and the traffic jam is a common thing. With the increasing rate of the traffic jam, the amount of wasted energy due to braking is also increasing. Hence, this technology can help to reduce the waste energy from electric vehicles in the world.

2. Methodology

The Regenerative braking systems vastly improve the fuel economy and further enhance the attractiveness of the vehicle. There are so many methods of regeneration of energy from braking such as (a) Electric Regenerative braking system (RBS), (b) Hydrostatic Regenerative Braking (HRB), (c) Regenerative Braking Using Compressed air and (d) Regenerative Braking Using Nitinol Spring. [24].

- a. The electric motor is the main driving components or regenerative braking system where electric motor acts as a motor as well as a generator.
- b. Another system called Hydrostatic Regenerative Braking (HRB) is brought in to improve the vehicle fuel economy by using electrical/electronic components and hydraulics.
- c. The power produced from the Regenerative braking system could be applied to fill the air tank during the time of braking. By absorbing the kinetic energy (necessary for braking), using the same for compressing the air and reuse these compressed air while powering the car.
- d. In the regenerative braking system using nitinol spring, the kinetic energy is stored in form of potential energy in spring. When the system actually demands the acceleration this potential energy stored is given back to the wheels to power them.

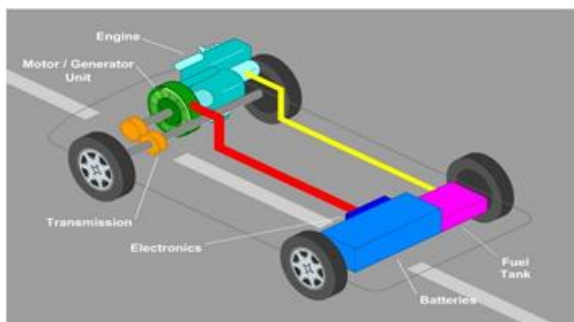


Fig. 1: Electric regeneration systems along with conventional IC engine.

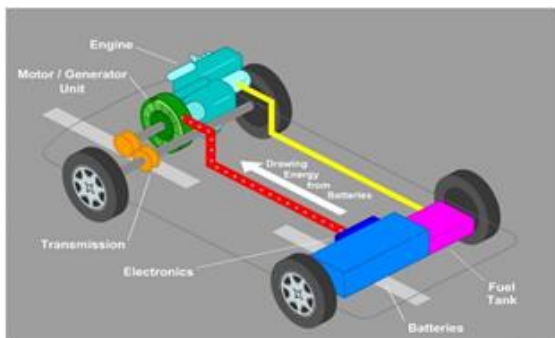


Fig. 2: Energy consumption from battery

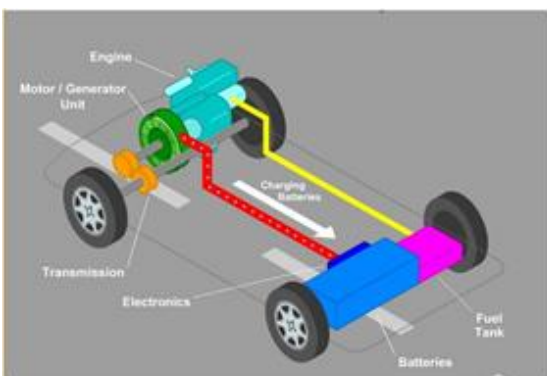


Fig. 3: Charging of battery when brake applied.

The electric regenerative system is selected over hydraulic regenerative system due to its less complexity, less noise, less weight and moreover the no chance of leakage. The regenerative braking system is more suitable for fully electric vehicles operating based on the electric motor. This electric motor expresses some interesting characteristics such as during the time of rotating in one way direction it converts the electrical energy into mechanical energy but when the electric motor moves in opposite direction it comes out as an electric generator. This energy aids to charge car batteries in performing various works. Once the car arrives at the speed then momentum is the main contributing factor to keeping it rolling. To accelerate the car from its idle condition energy from the battery is required, which is charged by the reverse motion of the electric motor.

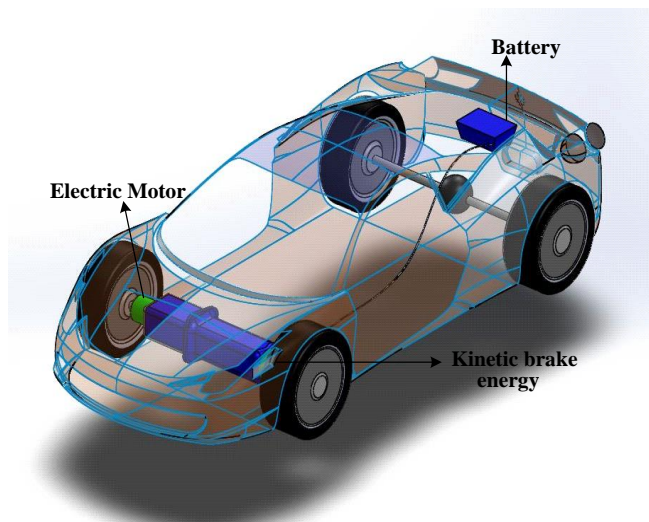


Fig. 4: The electric motor reverse direction, becoming a generator, which then stores the energy in the vehicle's battery.

This consequence is demonstrated in Figure 1, 2 and 3. An electric circuit is necessary to direct the electricity into battery produced by the motor. The regenerative braking system is illustrated in Figure 4 capable of converting the kinetic energy into electricity. The battery is then charged by this electricity.

3. Design and Construction

The main components of the designed electrical vehicle are an electrical controller, shaft-gear system, and wheels. The selection criteria and design method are discussed in the following sections.

3.1 Shaft Design

A mechanical device like the shaft is used to transmit power from the prime mover (electric motor or an engine) to the other rotating parts (Figure 5). The shaft stress analysis most of the time involves the combined stress approach based on the coincidental occurrences of torsional and normal stress due to bending. The distortion theory of failure mainly recommended for the shaft design and analysis. When an axial load is applied then the vertical shear stress and direct normal stress is obvious. But in the absence of the bending on a very short shaft or a short portion of a shaft, such stresses are dominant.

The amount of power available for 12V DC brush motor, $P = 1.25W = 0.0017 \text{ hp}$.

The torque transmitted by the gear is calculated by using formula

$$T = \frac{6300hp}{n} \quad (1)$$

For material AISI C1050 the endurance strength, yield strength, shears endurance strength are calculated from the formula (2), (3), (4) respectively [25]

$$S_n = 0.5S_u \quad (2)$$

$$S_{ys} = 0.6S_y \quad (3)$$

$$S_{ns} = 0.6S_n \quad (4)$$

The flexure or bending stress [26] is calculated from the formula (5)

$$S_a = \frac{Mc}{I} \quad (5)$$

Here $\frac{c}{I}$ is known as section modulus.

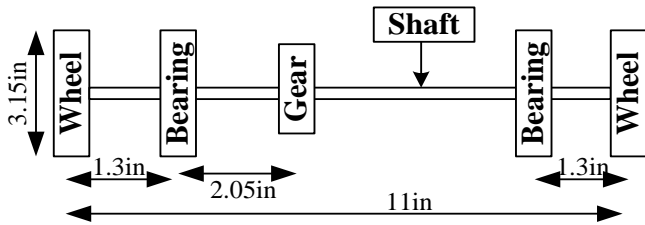


Fig. 5: Location of wheels, bearing, and gear on the shaft.

The equivalent stress and static shear stress [25] developed on the shaft is found from the formula (6), (7) and, (8) respectively

$$S_e = \frac{S_n}{S_y} S_m + K_f S_a \quad (6)$$

$$S_{ms} = \frac{Tc}{J} \quad (7)$$

$$S_{es} = \frac{S_{ns}}{S_{ys}} S_{ms} + K_{fs} S_{as} \quad (8)$$

The equation of combined variable stress [25] is

$$\frac{1}{N} = \left[\left(\frac{S_e}{S_n} \right) + \left(\frac{S_{es}}{S_{ns}} \right)^2 \right]^{\frac{1}{2}} \quad \text{[Here, N is the size factor]} \quad (9)$$

From the above equation finally shaft diameter is calculated. The shaft diameter = 0.3 in = 0.76 cm

3.2 Gear Design

Standard tool causes gear design quite simple, economical, reducing tooling expenses and inventory. Gears of the standard parameter are always preferable because of its low noise and vibration, high density of power transmission. Geometrical accuracy with metallurgical integrity is important for any highly stressed gear design. Gear designs to reduce the contact temperature during the time of high speed, bending stress minimization, balancing strength, balancing durability, balancing reliability. The gear used in the project is designed based on the following steps.

At first, the Pitch line speed is determined by using the equation, $V_m = \frac{\pi D_p N_g}{12}$ (10)

As $fpm < 2000$ fpm, commercially cut gears should be used [26].

Then the transmitted load and the dynamic load [26] is calculated for the commercially cut gear

$$F_t = \frac{33000hp}{V_m} \quad (11)$$

$$F_d = \frac{600 + V_m}{600} \times F_t \quad (12)$$

$$\text{For, Material Cast iron class 20, } S = 0.4S_u \quad (13)$$

According to ref. [25],

$$K_f = 0.4, Y = 0.289$$

$$\text{Diametric pitch is } P_d = \frac{N_g}{D} \quad (14)$$

$$\text{So, the Lewis equation, } F_s = \frac{SbY}{K_f P_d} \quad (15)$$

Then the following parameters are obtained

$$\text{Addendum, } a = \frac{1}{P_d}, \text{ Dedendum, } d = \frac{1.25}{P_d}$$

$$\text{Clearance} = \frac{0.25}{P_d}, \text{ Working depth} = \frac{2}{P_d}$$

$$\text{Whole depth, Outside diameter} = D + 2a$$

At last, the gear ratio is obtained, which tends to 14. But Pinion of 14 teeth is not available everywhere, so for the convenience of the work, a pinion of 15 teeth is selected.

3.3 Wheel Design

The most basic that comes to mind is often wheel diameter and wheel width. Wheel diameter is important because it controls the speed; wheel width helps to control ground pressure. Flange angle should be considered as high as possible and high contact stress should be avoided during wheel design.

Since the Height of the shaft from the base is 7 cm.

So the radius of the wheel should be less than 0.07 m. for this reason radius of the wheel is taken 0.043 m.

$$\begin{aligned} \text{Total amount of load} &= 2 \text{ set} \times (\text{metal wheel} + \text{thicker} \\ &\quad \text{wooden wheel} + \text{thinner wooden wheel}) \\ &= 648\text{gm} \end{aligned}$$

$$\text{Total no of wheel} = 6$$

$$\text{No. of metal wheel} = 2$$

$$\text{No. of thicker wooden wheel} = 2$$

$$\text{No. of thinner wooden wheel} = 2$$

$$\text{We know, } \rho = \frac{m}{V} \text{ or } m = \rho \times V$$

$$m = \rho \times \pi r^2 t \text{ or } t = \frac{m}{\rho \pi r^2} \quad (16)$$

Density of low carbon steel (0.2% C) is 7850kg/m³

For metal wheel, thickness (t) is 0.005 m or 0.075in

For thicker wooden wheel, thickness (t) is 0.025m or 1in

For thinner wooden wheel, thickness (t) is 0.011m or 0.45in

After proper scrutiny, all the elements are assembled on the wooden structure. Figure 6 indicates the pictorial view of the prototype.

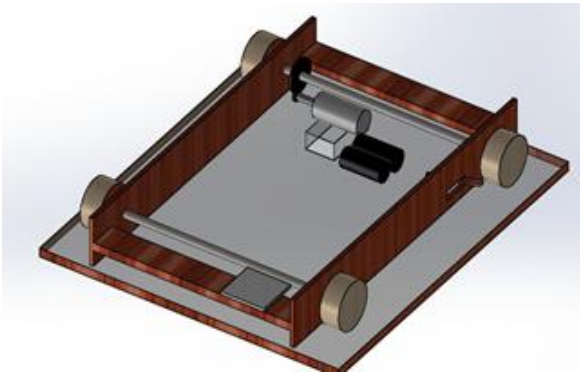


Fig. 6: Pictorial view of the prototype.

4. Experimental Setup

The experimental test rig is illustrated in Figure 7. It consists of a motor with transformer, relay, and capacitor to store the charge into the battery. The measurement device used in this experimental test is a tachometer for RPM, a stopwatch for a time, ammeter and voltmeter for measuring voltage and current and VU meter for checking the signal condition. The tachometer accuracy is $\pm 0.05\%$. When power is supplied, shaft starts rotating with the rotation of the motor. When the brake is applied, due to inertia the shaft continued to rotate for few seconds. As the gear and pinion mesh with the rotation of the shaft will operate the motor and motor works as a generator producing electricity, which is stored in the battery.

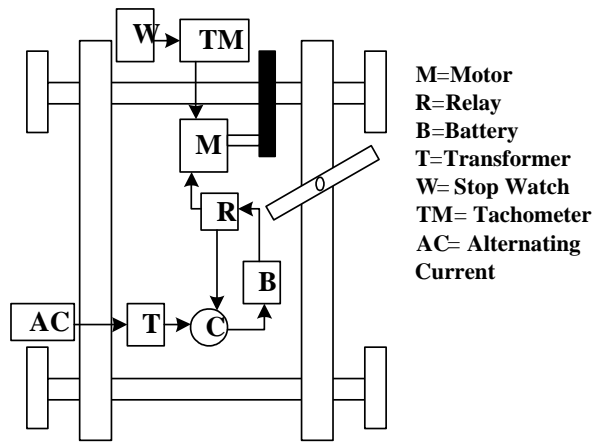


Fig.7: Schematic view of power transmission from AC voltage to the motor.

Figure 7 represents the schematic view of power transmission in which a gear is fitted to the shaft on which the load is mounted. A pinion mounted on the motor shaft is meshed with the gear. So, when power is supplied to the motor, the shaft started rotating, so the loads e.g. wheels.

220V AC connections are used as power source. To lower down the voltage to 2x12V a single phase center tapped transformer is used. The single phase system consists of neutral wire allowing the system to use a higher voltage while still supporting lower voltage single phase loads. The

other two wires provided 12V each. Then the neutral wire is connected to one end of the capacitor and the rest two

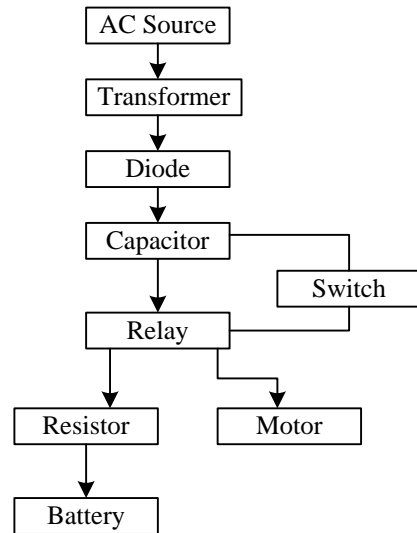


Fig. 8: Flow chart of power from the main source to the motor.

connected to another end through the diode. As the diodes acted as the rectifier, they ensure that after braking no current comes back to the transformer. The capacitor acts as a very a close source of power. So, high-speed power could be pulled from the capacitor and the power source slowly charged the capacitor. Now five connections left from the capacitor. The end, at which 12V wires are connected, also connected to the two legs of the relay. Other three connections go to the positive port of battery, positive port of motor and to the braking switch. The relay, which is used consisted of five legs. Two legs are connected to the capacitor, one to the negative end of the motor, one to the braking switch and the other one to the negative port to the battery. A resistor is employed between the relay and battery to drop the excess current for the betterment of the battery health. A VU meter across the resistor showed the current drop. Figure 8 presents the supply of power from the source to motor whereas Figure 9 points out the complete electric circuit connection of the prototype.

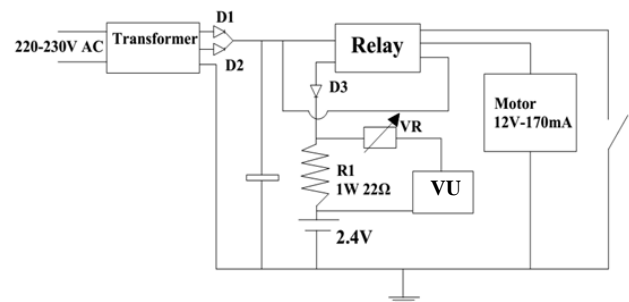


Fig. 9: Electric circuit showing the connection between electric elements.

5. Experimental Results and Discussions

To operate the prototype 220V AC as a power source is used. Then a single phase transformer is used, which produced 24V. A capacitor of 470μF and a resistor of 22Ω are used to store the regenerative energy.

The system is tested with regeneration applying variable loads. In this study, among the two sets of the motor wheel, 3 wheels mass 36, 83 and 205g are used. For the first observation, one set of wheels are mounted on one end and the other set of wheels are mounted on the other end of the driving shaft. So the total load becomes 648g. For this load RPM, time of rotation, voltage, current, charge is measured. Then another combination of 205g and 83g are loaded on the driving shaft. So this time total load becomes 576g. Similarly, when a combination of 205g, 36g, and 83g are used total load becomes 482g and 238g respectively. When single loads are used load becomes 166g. Using Voltmeter, the voltage produced in the battery terminal after braking=2.4V. Using Ammeter the current measured entering the battery immediately after braking = 200mA.

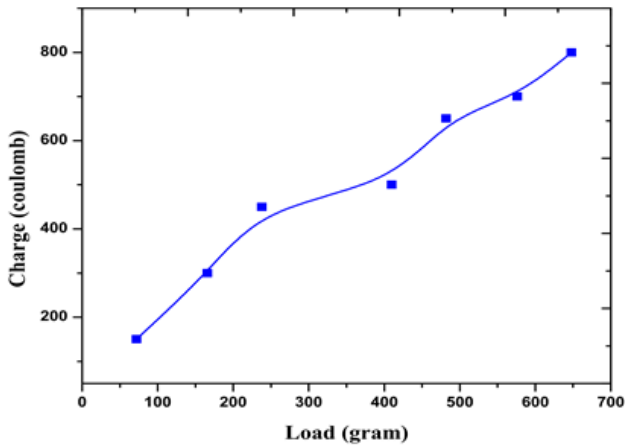


Fig. 10: Variation of the charge storing with the applied braking load.

For various load conditions, the prototype is tested and the amount of charge stored is determined. The battery is recharged by 0.80C when the maximum load is 648g and for the minimum load 72g, the battery is recharged by 0.15C. Figure 10 illustrates that the charge increases with the increasing load.

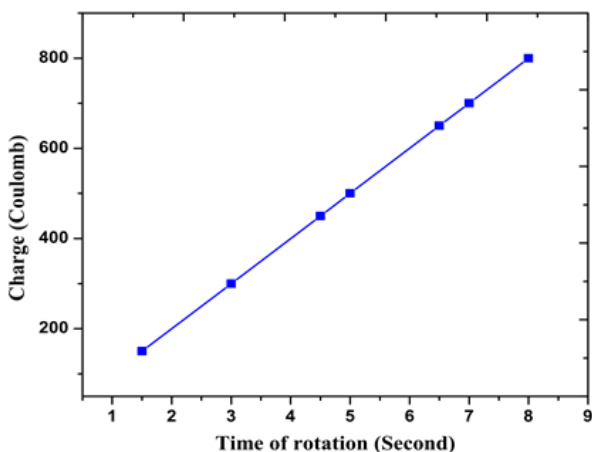


Fig. 11: Variation of the charge storing with the time of rotation after braking.

The amount of charge depended on the time of rotation of shaft after braking. More time offered more charge to store in the battery. The maximum time of rotation is 8sec at which the battery is recharged by 0.80C and the battery is recharged

by 0.15C for the minimum time of rotation 1.5sec. Figure11 shows a straight line in increasing order of charge with increasing the time of rotation.

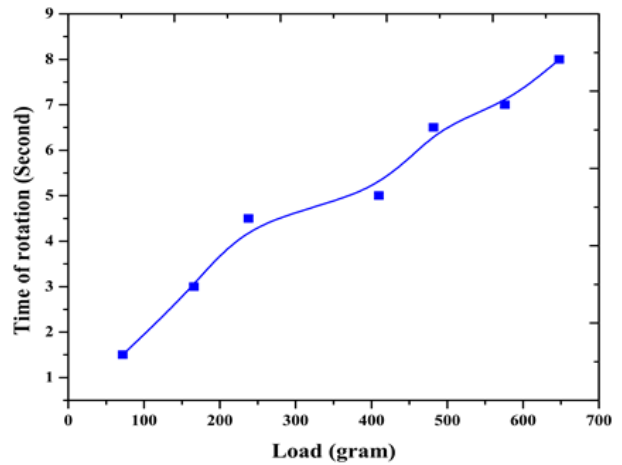


Fig. 12: Variation of time of rotation after braking with the applied braking load.

Property of inertia is to keep moving object in motion. So, at the time of braking, this inertia tends to make the vehicle in motion. The load had its direct impact on inertia. The maximum load 648g allowed the wheel to rotate for 8sec after braking and at the minimum load 72g, the time of rotation is 1.5 sec. So the time of rotation directly depended upon load as shown in Figure 12. From Figure 12, it is also demonstrated that the time of rotation is also increasing with the increasing load.

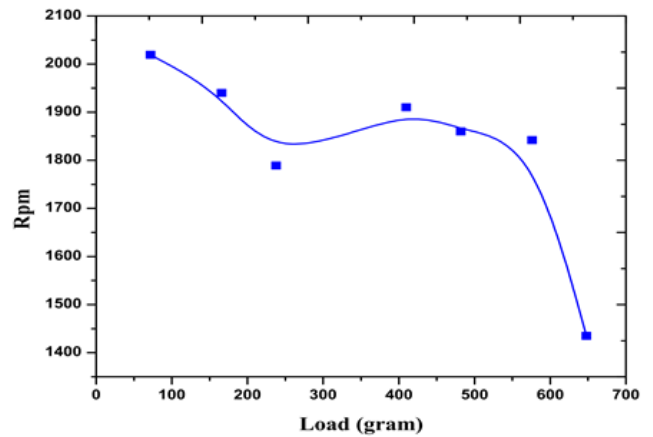


Fig. 13: Variation of RPM of the shaft with the applied braking load.

Load on driving shaft affected the RPM of the wheel. Figure 13 exemplifies that when the load is 72g, the speed is a maximum of 2010RPM and gradually decreases to a minimum of 1435RPM when the load is 648g.

When the amount of current forced through the wire is limited to 200mA with a 22Ω resistor. Then the power, $P = 4.2 \times 0.2 = 0.84W$

The energy (E) stored in a battery is equal to,

$$E = QV = i \times t \times V = P \times t \tag{17}$$

Where for the different braking load the charge (Q) is determined from the formula, $Q = it$.

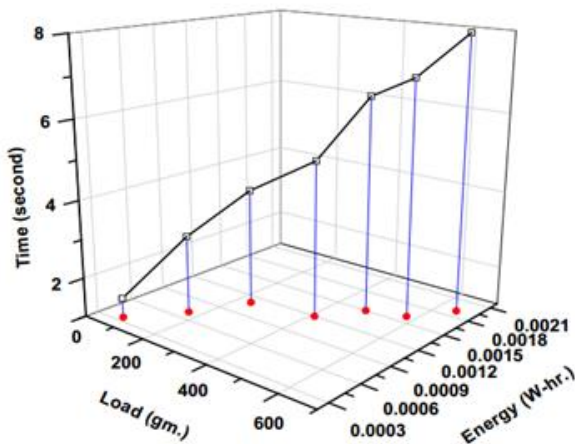


Fig. 14: Load vs. Energy vs. time curve.

Figure 14 illustrates the relation between the load, time of rotation of shaft after braking and the energy stored in the battery. Time of rotation of shaft after braking is 1.5 sec for the minimum load 72g. and the energy stored is 0.00035 W-hr. The shaft is rotating for 8sec for the maximum load 648g and 0.0019W-hr. energy is stored. More time of rotation resulted in more energy stored in the battery. The continuous increasing of energy with the increasing load and time is shown in Figure 14.

Economic evaluation of the prototype electric regenerative system is made done based on the Installation cost and the cost of the equipment used in the study. However, these costs are estimated based on regarding the local market price. Only fixed cost is considered, which consists of annual maintenance cost and replacement cost. The maintenance cost is expected to grow annually whereas the replacement cost is considered constant throughout the five-year life span. The fixed cost compared to operation period is very less. Here, the operation related labor cost is considered as negligible. Installation of the regenerative system in the vehicle can play a supportive role for the use of auxiliary elements such as lights, fans etc. Finally, to identify the project profitability calculation of benefit to cost ratio and sensitivity analyses are carried out.

6. Conclusion

In this study, a prototype experimental electrical vehicle is successfully designed and tested to study the energy regeneration for breaking load. Analyzing the results, the following conclusions are drawn:

- a. The battery is recharged by 0.80C at the maximum braking load of 648g. At the minimum braking load of 72g. the battery is recharged by 0.15C. It is investigated that, the amount of energy stored is increased with the increasing of the load.
- b. The speed of the shaft is 1435RPM at the maximum load and the speed of the shaft is 2019 RPM at the minimum load. The study also shows that the RPM is inversely proportional to the braking load.

This study presented the proper potency of the regenerative braking system in case of utilizing the waste energy. A brushless dc motor is recommended as an efficient choice for the further future work.

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References

1. Longo, M., et al., Replacement of vehicle fleet with EVs using PV energy. *International Journal of Renewable Energy Research (IJRER)*, 2015. 5(4): p. 1146-1153.
2. Demirbas, A. and G. Arin, An overview of biomass pyrolysis. *Energy sources*, 2002. 24(5): p. 471-482.
3. Mohan, D., C.U. Pittman, and P.H. Steele, Pyrolysis of wood/biomass for bio-oil: a critical review. *Energy & fuels*, 2006. 20(3): p. 848-889.
4. Wicks, F. and K. Donnelly. Modeling regenerative braking and storage for vehicles. in *Energy Conversion Engineering Conference, 1997. IECEC-97., Proceedings of the 32nd Intersociety.* 1997. IEEE.
5. Gao, Y., L. Chen, and M. Ehsani, Investigation of the Effectiveness of Regenerative Braking for EV and HEV, 1999, SAE Technical Paper.
6. Bildstein, M., K. Mann, and B. Richter, Regenerative braking system, in *Fundamentals of Automotive and Engine Technology.* 2014, Springer. p. 240-243.
7. Ahn, J., et al., Analysis of a regenerative braking system for hybrid electric vehicles using an electro-mechanical brake. *International Journal of Automotive Technology*, 2009. 10(2): p. 229-234.
8. Peng, D., et al., Combined control of a regenerative braking and antilock braking system for hybrid electric vehicles. *International Journal of Automotive Technology*, 2008. 9(6): p. 749-757.
9. Cao, J. and A. Emadi, A new battery/ultracapacitor hybrid energy storage system for electric, hybrid, and plug-in hybrid electric vehicles. *IEEE Transactions on power electronics*, 2012. 27(1): p. 122-132.
10. Cikanek, S. and K. Bailey. Regenerative braking system for a hybrid electric vehicle. in *American Control Conference, 2002. Proceedings of the 2002.* 2002. IEEE.
11. Buecherl, D., et al., Derivation of Basic Energy Storage Parameters for Future Electric Vehicles. *International Journal of Renewable Energy Research (IJRER)*, 2011. 1(2): p. 105-109.
12. Kutsmeda, K.J., K.G. Fehrle, and P.J. Trick. Computer modeling, simulation, and validation by field testing of a traction power system for electric trolley buses. in *Railroad Conference, 1995., Proceedings of the 1995 IEEE/ASME Joint.* 1995. IEEE.
13. Clegg, S., A review of regenerative braking systems. 1996.

14. Yeo, H. and H. Kim, Hardware-in-the-loop simulation of regenerative braking for a hybrid electric vehicle. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 2002. 216(11): p. 855-864.
15. Gao, H., Y. Gao, and M. Ehsani, Design issues of the switched reluctance motor drive for propulsion and regenerative braking in EV and HEV, 2001, SAE Technical Paper.
16. Michaelis, L. and O. Davidson, GHG mitigation in the transport sector. Energy Policy, 1996. 24(10): p. 969-984.
17. Kreutz, S. Ideal regeneration with electromechanical Brake Booster (eBKV) in Volkswagen e-up! and Porsche 918 Spyder. in 5th International Munich Chassis Symposium 2014. 2014. Springer.
18. Wang, X., et al., Application study on the dynamic programming algorithm for energy management of plug-in hybrid electric vehicles. Energies, 2015. 8(4): p. 3225-3244.
19. Zheng, P., et al., Investigation of a novel 24-slot/14-pole six-phase fault-tolerant modular permanent-magnet in-wheel motor for electric vehicles. Energies, 2013. 6(10): p. 4980-5002.
20. Khaligh, A. and Z. Li, Battery, ultracapacitor, fuel cell, and hybrid energy storage systems for electric, hybrid electric, fuel cell, and plug-in hybrid electric vehicles: State of the art. Vehicular Technology, IEEE Transactions on, 2010. 59(6): p. 2806-2814.
21. Brenna, M., F. Foadelli, and M. Longo, The exploitation of vehicle-to-grid function for power quality improvement in a smart grid. IEEE Transactions on Intelligent Transportation Systems, 2014. 15(5): p. 2169-2177.
22. Lin, C.-C., et al., Power management strategy for a parallel hybrid electric truck. IEEE transactions on control systems technology, 2003. 11(6): p. 839-849.
23. Tsotoulidis, S.N. and A.N. Safacas, Analysis of a drive system in a fuel cell and battery powered electric vehicle. International Journal of Renewable Energy Research, IJRER, 2011. 1(3): p. 140-151.
24. KUMAR, H., REGENERATIVE BRAKING.
25. M, F.V., "Design of Machine Element", the MACMILLAN Company, New York/ COLLIER-MACMILLAN Ltd, London. : Printed in United States of America. p. p-242-243, p-358,366-369.
26. Pytel A., S.F.L., "Strength of Materials" HarperCollins Publishers: Singapore.