State and Parameter Estimation of Power Systems using Phasor Measurement Units as Bilinear System Model

Rajnish Kumar *, Michael G Giesselmann **‡, Miao He ***

* Department of Electrical & Computer Engineering, PhD, Texas Tech University, Lubbock TX

** Department of Electrical & Computer Engineering, Prof & Dept. chair, Texas Tech University, Lubbock TX

*** Department of Electrical & Computer Engineering, Assistant Prof, Texas Tech University, Lubbock TX

(rajnish.kumar@ttu.edu, michael.giesselmann@ttu.edu, miao.he@ttu.edu)

‡ Corresponding Author; Rajnish Kumar, 3002 4th street apt C200 Lubbock TX 79415, Tel: +1 806 445 3304, Rajnish.kumar@ttu.edu

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Abstract- In this paper, the development of a Parallel Kalman Filter for a bilinear model in the presence of correlation between process and measurement noise is discussed. The developed theory is implemented for estimation of both states and parameters of power system networks using measurements from synchronized Phasor Measurement Units (PMUs). Dynamics of states and parameters of power system networks comprise system dynamics for the bilinear system model representation, and measurements coming from PMUs are represented as observation for bilinear system model. Correlation between noise in voltage phasor dynamics and noise in PMU measurements as well as correlation between noises in parameter dynamics and measurement noise are considered for implementation of state and parameter estimation of power systems. The developed theory is implemented as a method to estimate voltage phasors and network parameters in parallel with each other. The developed theory is tested on various example power grids to show the effects of relevant correlation matrices on estimation of system states and network parameters of the power system network.

Keywords State Estimation, Parameter Estimation, Parallel Kalman Filter, Phasor Measurement Units, Power Systems.

1. Introduction

Most of the decisions related to secure and economic operations of the grid are dependent on the outcome of state estimator of power systems [1, 2]. It is the central block in an Energy Management System (EMS), which gives inputs to both Energy/economy functions subsystems and security monitoring and control subsystems [3]. With the complexity in modern grid increasing exponentially with time, the role of state estimator has become even more important for secure operation of the power grid. Phasor measurement units provide measurements at much faster rate compared to traditional Supervisory Control and Data Acquisition system (SCADA). Apart from this, measurements from PMUs are synchronized with GPS clock and there is a linear relationship between measurements and system states in state space representation, making the analyses simpler. PMUs work according to a defined IEEE standard to ensure compatibility with relevant communication protocols [4].

Schweppe first introduced application of static state estimation technique to state estimation of power systems [5-7]. The noisy measurements coming from SCADA from one time instant were used to estimate voltage magnitudes and phase angles at every bus using weighted least squares method. Later on after introduction of PMUs, various researchers solved problems of incorporating SCADA and PMU measurements into the state estimation problem [8-9]. In [10] authors implemented a state estimator with PMUs in case of presence of phase biasing, which is a special form of bad data present in PMU measurements. The estimator was implemented on a small section of American Electric Power (AEP) grid. In [11] a distributed state estimator was implemented using PMU measurements. In this case, the estimator is divided into smaller local state estimators, each running their own estimation process. The overall effect is estimator being more robust because of its decentralized nature.

Dynamic state estimation (DSE) techniques use information from both system dynamics and observer model to estimate system states. DSE has been used to estimate states of power system networks in various ways. A method to improve performance of DSE is developed in [12] which is extended from the model used in [13]. In [14], DSE is implemented in case of sudden load change using two different methods. The first method involves an iterative estimation at the instant of load change while the second method decreases effects of nonlinearity due to sudden change in load by using Taylor series expansion up to second term. It is found that a lack of synchronization in GPS clocks used by PMUs can deteriorate the performance of the state estimator. In [15], lack of synchronization is included in the estimation process. Both state dynamics model and synchronization error model are included into a bilinear model and state estimation is implemented using Alternating Minimization technique and Parallel Kalman Filter technique.

Correct implementation of state estimator depends on correct knowledge of the parameters of the network. Despite of state estimation being a very active research topic and methods in state and parameter estimations being similar, not much effort has been directed towards parameter estimation of power systems. In [16] a method to estimate parameter errors in power system networks was developed which uses real time measurements. The algorithm uses weighted least squares for state estimation part while using sensitivity analyses for error identification. In [17-18], authors have introduced a method to implement joint estimation of states and parameters of power system networks using synchrophasor measurements when the correlation between errors in state prediction and pseudo measurement errors is known. Similar techniques have been used for both states and parameter estimation. Researchers have relied on various methods to implement parameter estimation using residual sensitivity analyses [19-21] and augmentation of states and parameters. Approaches based on normal equations [22-23] have been used which are extension to conventional state estimator in a manner that it is an augmentation of states and parameters of the network as a representation of new system states for the estimator. These methods may face issues related to convergence and observability. Power system network parameter estimation has been implemented in various ways based on dynamic state estimation techniques [24-26]. Approaches based on dynamic state estimation technique use information from both dynamics of parameters as well as measurements coming from various parts of the network and tend to be computationally expensive. In [28-38] some other important ways to implement state estimation of power systems have been discussed.

Current methods for state/parameter estimation of power systems don't take correlation between noises of system dynamics and measurements in to account. In this paper a new method for estimation of state-parameter bilinear system using Parallel Kalman Filter is developed for a case when correlation between various sections of process noise and measurement noise is present. PKF is a dynamic state estimation technique for a special class of nonlinear systems where the system states can be separated into two set of vectors each being a linear system. Derivation of PKF is based on game theory approach where decisions made by each team is in terms of the outcome of decisions made by the other team and vice versa [27]. Dynamics of voltage phasors and network parameters along with PMU measurements is combined into a format of bilinear system model. The correlation between noise in process dynamics of voltage phasors and parameter networks with noise in PMU measurements is known.

The developed algorithm is used for estimation of system states and network parameters of power systems using PMU measurements. Developed algorithm is implemented on various standard bus test systems, e.g. IEEE 14, IEEE 30, IEEE 57 and IEEE 118 bus test systems and effects of correlation matrices on state/parameter estimation is verified using Root Mean Square Error values. Numerical Results reveal the effectiveness of the proposed method.

2. Measurement and System Model

2.1 Measurement Model

For an example power system network consisting of *N* nodes connected by *L* transmission lines, the system states at time instant *k* are given by

$$X_k = \begin{bmatrix}|V_1| & |V_2| \ldots \ldots |V_N| & \theta_2 & \theta_3 & \theta_N\end{bmatrix}_k^T \quad (1)$$

This system state representation can be changed to rectangular coordinates without any loss of accuracy, and is given by

$$X_k = \begin{bmatrix}X_1^i & X_2^i \ldots \ldots X_N^i & X_1^r & X_2^r \ldots \ldots X_N^r\end{bmatrix}_k^T \quad (2)$$

Where, X_1^i and X_1^r are imaginary and real part of voltage phasor *X* for bus *1* and so on for time instant *k*. Relationship between branch currents with voltage phasors in the power systems has been derived using *pi model* of the transmission lines.

Fig. 1 Pi model of the transmission line [1]

Parameters of line *i-j* connecting nodes *i* and *j* are given by conductance g_{ij} and susceptance b_{ij}. Shunt conductance g_{si} is assumed to be negligible, b_{si} represents shunt susceptance for node *i*. Admittance of the line in fig. 1 is given by

$$Y_{ij} = g_{ij} + j\, b_{ij} \quad (3)$$

Voltage phasors at nodes *i* and *j* and current phasors through line *i-j* are related as follows

$$I_{ij,k}^r = g_{ij,k}.X_{i,k}^r - g_{ij,k}.X_{j,k}^r - (b_{ij,k} + b_{sh,k}).X_{i,k}^i + b_{ij,k}.X_{j,k}^i \quad (4)$$

$$I_{ij,k}^i = g_{ij,k}.X_{i,k}^i - g_{ij,k}.X_{j,k}^i + \left(b_{ij,k} + b_{sh,k}\right)X_{i,k}^r - b_{ij,k}\, X_{j,k}^r \quad (5)$$

Here it should be noted that expressions for real and imaginary components of current phasors are given in terms of voltage phasors and network parameters, all of which vary with time instant k. The set of measurement equations for PMU placed at both node i and j can be given as

$$Z_k = H_k X_k \quad (6)$$

Where, subscript k denotes time instant for the measurement set Z_k.

For a small subnetwork as given in fig. (1), measurement set Z_k (which is a combination of voltage phasors due to PMUs at node i and j and current phasors through the line i-j) at time instant k is given by

$$Z_k = \left[\, X^r_{i,k} \;\; X^r_{j,k} \;\; X^i_{i,k} \;\; X^i_{j,k} \;\; I^r_{ij,k} \;\; I^r_{ji,k} \;\; I^i_{ij,k} \;\; I^i_{ji,k} \right]^T \quad (7)$$

For a set of PMUs placed on a network, each types of measurements generated can be combined together to represent the measurement set as in (6). The measurement Jacobian matrix H_k in (6) can be given as

$$H_k = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \\ H_{31} & H_{32} \\ H_{41} & H_{42} \end{bmatrix} \quad (8)$$

Where, each element of the Jacobian matrix represents the rate of change of corresponding type of measurement to a subset of system states (X_i or X_r) as shown for elements in first row

$$H_{11} = \frac{\partial X^r_k}{\partial X^i_k} \qquad H_{12} = \frac{\partial X^r_k}{\partial X^r_k}$$

It is assumed that errors associated with PMU measurements are of Gaussian distribution in nature with zero mean and known covariance. Thus the PMU measurements can be modeled as

$$Z_k = H_{X,k} X_k + W_{Z,k} \quad (9)$$

Here $W_{Z,k}$ represents PMU measurement noise with Gaussian distribution represented by

$$W_{Z,k} \sim \aleph(0, R_k) \quad (10)$$

It should be noted that the elements in Jacobian matrix $H_{X,k}$ consists of time varying parameters of the power system network.

2.2 System State Dynamics

Following the state space representation from [17], we represent dynamics of system space in state space form where state vectors change only slightly around a central value. This representation is accurate in case of voltage phasors in power systems because voltage phasors in a balanced power system network don't vary too far from a central value.

$$X_k = X_c + W_{x,k} \quad (11)$$

Random vector $W_{x,k}$ takes a multivariate Gaussian distribution with zero mean and known covariance.

$$W_{X,k} \sim \aleph(0, Q_{X,k}) \quad (12)$$

2.3 Network Parameter Dynamics

Parameters of the power system network lines are modeled according to pi model of the transmission lines. Parameter Y for a transmission line connecting buses i and j is a vector consisting of gij, bij, and bsh associated with line i-j in our representation of parameter vectors. We represent dynamics of parameters of power systems network in state space model as follows [17]

$$Y_k = Y_c + W_{Y,k} \quad (13)$$

Where, $W_{y,k}$ represents noise in parameter dynamics which is assumed to be of Gaussian distribution as

$$W_{Y,k} \sim \aleph(0, Q_{Y,k}) \quad (14)$$

Using the known dynamics of network parameters, above measurement coming from PMUs can be represented in a new format given as follows

$$Z_k = H_{Y,k} Y_k + W_{Z,k} \quad (15)$$

Where, V_k represents random noise with Gaussian distribution associated with PMU measurements described in (10). Elements of Jacobian matrix $H_{Y,k}$ used in (15) will be a linear function of system state vectors X^r_k or X^i_k and can be expressed as

$$H_{Y,k} = \begin{bmatrix} HY_{11} & HY_{12} & HY_{13} \\ HY_{21} & HY_{22} & HY_{23} \\ HY_{31} & HY_{32} & HY_{33} \\ HY_{41} & HY_{42} & HY_{43} \end{bmatrix} \quad (16)$$

Elements of first row of Jacobian matrix in representation of second form of PMU measurements are given as follows

$$HY_{11} = \frac{\partial X^r_k}{\partial\, B\_line^i_k} \qquad HY_{12} = \frac{\partial X^r_k}{\partial\, G\_line^i_k} \qquad HY_{13} = \frac{\partial X^r_k}{\partial\, B\_shk}$$

It is assumed that there is correlation between process and measurements noises in the system. Which means noise in system dynamics of voltage phasors $W_{X,k}$ and noise in measurements i.e. $W_{Z,k}$ are correlated and their correlation is given by M_{XZ}. Similarly, noise in process dynamics of parameters $W_{Y,k}$ and noise in measurements are correlated and their correlation is represented by matrix M_{YZ}.

3. Development of PKF for Bilinear Model

In this section, details of PKF is discussed for a case when correlation between process and measurement noise is present. Development of the theory behind this is discussed in appendix A. PKF for bilinear system was first developed in [27]. A bilinear system is a special class of discrete time nonlinear system where given a partition in state vector, the system can be represented into two separate linear systems with respect to the system states of their own partition. This representation can be very useful for state estimation when system states are in form of a linear function of unknown parameters. For a hypothetical system, let us assume that the state vector can be partitioned into two vectors namely, X_k and Y_k (in accordance with states and parameters in case of power systems). The partitioned vectors are assumed to be of dimensions such as $X_k \in \mathcal{R}^p$ and $Y_k \in \mathcal{R}^q$. In this case, the dynamics of the states can be represented as

$$\begin{pmatrix} X_{k+1} \\ Y_{k+1} \end{pmatrix} = \left(\frac{A_{11} + F_1(Y_k)}{A_{21}} \middle| \frac{A_{12}}{A_{22} + F_2(X_k)} \right) \begin{pmatrix} X_k \\ Y_k \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} u_k + \begin{pmatrix} \varepsilon_{k1} \\ \varepsilon_{k2} \end{pmatrix} \quad (17)$$

Where, ε_{k1} and ε_{k2} are zero mean white noise with known covariance matrices Q_{k1} and Q_{k2} respectively. For power systems, X_k represents the voltage phasors and Y_k represents network parameters. In above model, $F_1(Y_k)$ and $F_2(X_k)$ depend linearly on Y_k and X_k respectively. This can be represented as

$$F_1(Y_k) = \sum_{i=1}^{q} F_{1i} Y_{ki} \quad (18)$$

$$F_2(X_k) = \sum_{i=1}^{p} F_{2i} X_{ki} \quad (19)$$

Two representations for observer model of the bilinear system are shown below

$$Z_k = H_1 X_k + H_2 Y_k + C_1(Y_k) X_k + V_k \quad (20)$$

$$Z_k = H_1 X_k + H_2 Y_k + C_2(X_k) Y_k + V_k \quad (21)$$

In the observer model shown above, $Z_k \epsilon \mathcal{R}^r$ is the measurement vector, functions $C_1(Y_k)$ and $C_2(X_k)$ are linear combinations of their respective arguments and V_k is zero mean white measurement noise with known covarianceR_k.It is assumed that the correlation between X and Z as well as correlation between Y and Z is known and is denoted by M_{XZ} and M_{YZ} respectively.

Initialization step of the PKF is same as a regular Kalman filter for each of the filters. Recursive steps for the implementation of DSE for each of the partition of state vectors in form of prediction and correction steps are given as follows

The first Kalman Filter for estimation of system states X_k is implemented as follows

Prediction:

$$\hat{X}_{k+1|k} = [A_{11} + F_1(\hat{Y}_{k|k-1})]\hat{X}_{k|k} + A_{12}\hat{Y}_{k|k-1} + B_1 u_k \quad (22)$$

$$P_{k|k-1}^{(1)} = [A_{11} + F_1(\hat{Y}_{k|k-1})] \; P_{k-1|k-1}^{(1)} \, [A_{11} + F_1(\hat{Y}_{k|k-1})]^T + Q_{k1} \quad (23)$$

Update:

$$S_k^{(1)} = \left(H_1 + C_1(\hat{Y}_{k|k-1})\right)^T P_{k|k-1}^{(1)} \left(H_1 + C_1(\hat{Y}_{k|k-1})\right) + R_k + \left(H_1 + C_1(\hat{Y}_{k|k-1})\right) M_{XZ} + M_{XZ}^T \left(H_1 + C_1(\hat{Y}_{k|k-1})\right)^T \quad (24)$$

$$K_k^{(1)} = [P_{k|k-1}^{(1)} \left(H_1 + C_1(\hat{Y}_{k|k-1})\right) + M_{XZ}][S_k^{(1)}]^{-1} \quad (25)$$

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k^{(1)} \left[Z_k - \left(H_1 \hat{X}_{k|k-1} + H_2 \hat{Y}_{k|k-1} + C_1(\hat{Y}_{k|k-1})\right)\hat{X}_{k|k-1}\right] \quad (26)$$

$$P_{k|k}^{(1)} = P_{k|k-1}^{(1)} - K_k^{(1)} \left[H_1 + C_1(\hat{Y}_{k|k-1})\right]^T P_{k|k-1}^{(1)} \quad (27)$$

The second Kalman Filter for estimation of system states Y_k is implemented as follows

Prediction:

$$\hat{Y}_{k+1|k} = [A_{22} + F_2(\hat{X}_{k|k-1})]\hat{Y}_{k|k} + A_{21}\hat{Y}_{k|k-1} + B_2 u_k \quad (28)$$

$$P_{k|k-1}^{(2)} = [A_{22} + F_2(\hat{X}_{k|k-1})] \;\; P_{k-1|k-1}^{(2)} \, [A_{22} + F_2(\hat{X}_{k|k-1})]^T + Q_{k2} \quad (29)$$

Update:

$$S_k^{(2)} = \left(H_2 + C_2(\hat{X}_{k|k-1})\right)^T P_{k|k-1}^{(2)} \left(H_2 + C_2(\hat{X}_{k|k-1})\right) + R_k + \left(H_2 + C_2(\hat{X}_{k|k-1})\right) M_{YZ} \; M_{YZ}^T \left(H_2 + C_2(\hat{X}_{k|k-1})\right)^T \quad (30)$$

$$K_k^{(2)} = P_{k|k-1}^{(2)} \left(H_2 + C_2(\hat{X}_{k|k-1})\right) [S_k^{(2)}]^{-1} \quad (31)$$

$$\hat{Y}_{k|k} = \hat{Y}_{k|k-1} + K_k^{(2)} \left[Z_k - \left(H_1 \hat{X}_{k|k-1} + H_2 \hat{Y}_{k|k-1} + C_2(\hat{X}_{k|k-1})\right)\hat{Y}_{k|k-1}\right] \quad (32)$$

$$P_{k|k}^{(2)} = P_{k|k-1}^{(2)} - K_k^{(2)} \left[H_2 + C_2(\hat{X}_{k|k-1})\right]^T P_{k|k-1}^{(2)} \quad (33)$$

Above set of equations represent two Kalman Filters acting in parallel with output of one being input to the other and vice versa.

4. Numerical Examples

The theory developed above has been implemented on various example power system network models to do state (voltage phasors of different buses) and parameter (parameters of transmission lines represented by pi model) estimation of the power system network. A small example of 3 bus test system [17] has been used to elaborate application of developed PKF for various cases of state and parameter estimation of power system model. The developed algorithm has also been tested on IEEE 14, IEEE 30, IEEE 57 and IEEE 118 bus test system to evaluate performance of the estimators.

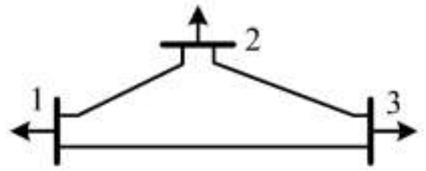

Fig. 2. 3 bus example network [17]

Dynamics of parameters as well as states along with various assumed correlations are presumed known. Parameters of 3 bus example network are given as follows.

Table 1 Parameters of example 3 bus system

Branch (m-n)	1-2	2-3	3-1
g_{mn}	2.5860	4.9196	2.1079
b_{mn}	-9.3535	-16.3529	-7.6040

Dynamics for system states were generated by adding white Gaussian noise of known covariance to the base case value according to (11). The system parameters are generated by adding white Gaussian noise of zero mean and known covariance into the base case value according to (13).

For first example, we discuss implementation of PKF for a case when PMUs are placed on nodes 1 and 2 of the example network. Values of system states generated for first 6 times instant according to dynamic model given in (11) is shown in Table 2

Table 2 System States for first 6 time instants

Time	**1**	**2**	**3**	**4**	**5**	**6**
X_1^i	0.0001	-0.0001	0	0	0	0
X_2^i	0.0401	0.0504	0.0417	0.0476	0.0477	0.0544
X_3^i	0.0753	0.0806	0.0788	0.0788	0.0781	0.0813
X_1^r	1.0132	1.0127	1.0106	1.0054	1.0128	1.0078
X_2^r	0.9846	0.9802	0.9833	0.9818	0.9829	0.9772
X_3^r	0.9968	1.0024	0.9934	0.9948	1.0023	0.9986

The covariance matrix for process noise is given by $Q_{x,k}$

$$Q_{xk} = \begin{matrix} 0.009 & -0.02 & -0.07 & -0.006 & -0.06 & 0.006 \\ -0.023 & 95.56 & -5.20 & -18.093 & 4.285 & 5.89 \\ -0.07 & -5.205 & 101.64 & -6.576 & 3.17 & 7.818 \\ -0.006 & -18.093 & -6.57 & 108.381 & -6.67 & -11.11 \\ -0.062 & 4.285 & 3.17 & -6.671 & 88.09 & -0.15 \\ 0.006 & 5.892 & 7.81 & -11.111 & -0.15 & 96.46 \end{matrix} \times 10^{-7}$$

The above covariance matrix is generated randomly for the process dynamics of system states. System dynamics were generated assuming the correlation between process noise of system states and measurement noise (represented by M_{XZ}). As discussed before, true values of system parameters were generated by adding white Gaussian noise of known covariance around a base case value of network parameters with including the effects of correlation matrix M_{YZ} just like system states. The covariance for process noise of network parameters for this case are given as follows

$$Q_{yk} = \begin{bmatrix} 10.08 & -0.47 & -0.29 & 0.27 & -0.96 & -0.97 \\ -0.47 & 7.21 & 0.87 & -0.47 & 0.66 & -0.03 \\ -0.29 & 0.87 & 9.64 & 0.45 & -0.24 & 0.92 \\ 0.27 & -0.47 & 0.45 & 10.43 & 0.81 & -0.09 \\ -0.96 & 0.66 & -0.24 & 0.81 & 10.07 & 1.06 \\ -0.97 & -0.03 & 0.92 & -0.09 & 1.06 & 9.56 \end{bmatrix} \times 10^{-6}$$

True values of system states and parameters were generated using covariance matrices Q_{Xk}, Q_{Yk}, M_{XZ}, M_{YZ} and R_k. Size of matrices M_{XZ}, M_{YZ} and R_k depend on number of PMUs placed on the network. Value of correlation between process noise of power system states and PMU measurements, denoted by M_{XZ} is given by $M_{xz} = [M_{xz1} \;\; M_{xz2}]$. Where M_{xz1} and M_{xz2} are as follows

$$M_{xz1} = \begin{bmatrix} -0.07 & 0.15 & -0.01 & 0.03 & -0.09 & -0.13 \\ -11.0 & -0.41 & -23.9 & -14.2 & -3.34 & 8.35 \\ 3.28 & -5.77 & -7.68 & 2.02 & -2.85 & 6.36 \\ -6.53 & 6.8 & -3.52 & -0.40 & -3.01 & -7.98 \\ 1.24 & -10 & -3.38 & -6.96 & 13.82 & -9.98 \\ -5.79 & -4.54 & 0.8 & -4.59 & 2.382 & -2.55 \end{bmatrix} \times 10^{-7}$$

$$M_{xz2} = \begin{bmatrix} 0 & 0.17 & -0.02 & -0.06 & -0.19 & -0.01 \\ -3.62 & -7.47 & -6.79 & -5.29 & 3.77 & 11.7 \\ -2.32 & 10.28 & 6.85 & -3.50 & -13.21 & 2.64 \\ -8.91 & -3.56 & 14.28 & -1.91 & -26.35 & -13.64 \\ 10.71 & -8.03 & -12.59 & 1.64 & 12.03 & -9.27 \\ -2.49 & 0.55 & 16.96 & -7.07 & -0.80 & -7.78 \end{bmatrix} \times 10^{-7}$$

Correlation between noise in parameter dynamics and PMU measurements, denoted by M_{YZ} is given as $M_{yz} = [M_{yz1} \;\; M_{yz2}]$.where M_{yz1} and M_{yz2} are given by

$$M_{yz1} = \begin{bmatrix} -2.38 & -3.46 & -12.06 & 9.19 & -6.08 & -1.26 \\ 10.65 & -8.73 & 1.41 & 0.005 & 0.73 & 0.64 \\ -6.10 & 6.31 & 1.74 & -1.27 & 4.33 & 15.01 \\ -25.4 & 9.76 & -7.09 & -6.67 & 13.38 & 14.29 \\ -3.88 & -6.58 & 35.80 & 3.03 & 1.88 & -7.29 \\ 1.79 & 7.83 & 11.86 & -10.33 & -1.47 & 7.71 \end{bmatrix} \times 10^{-7}$$

$$M_{yz2} = \begin{bmatrix} -10.93 & -3.31 & 18.88 & 22.96 & 4.03 & -3.64 \\ 1.55 & 7.94 & 7.45 & 4.28 & -11.49 & 2.03 \\ -9.72 & -3.26 & 9.77 & 16.12 & -2.14 & 1.83 \\ 2.12 & -14.25 & -8.97 & -6.51 & 1.28 & -6.52 \\ -2.01 & -4.18 & 7.37 & -9.53 & 22.39 & 3.14 \\ 8.77 & -11.93 & 3.94 & -10.39 & 6.62 & -10.32 \end{bmatrix} \times 10^{-7}$$

Root Mean Square Error (RMSE) has been used as a performance metric for both the state and parameter estimation filters for developed PKF. RMSE of estimated state $\hat{X}_{k|k}$ at time instant k using M Monte Carlo simulations is defined as

$$RMSE(\hat{X}_{k|k}) \approx \sqrt{\left(\frac{1}{M}\sum_{i=1}^{M}\tilde{X}_{k|k}^{T}(i)\tilde{X}_{k|k}(i)\right)} \tag{34}$$

Where $\tilde{X}_{k|k}(i)$ is the estimation error of state vector X at time instant k. Effects of wrong values of correlation matrices were demonstrated by implementing two Parallel Kalman filters for each of the cases. First PKF does both state and parameter estimation using correct values of correlation matrices M_{XZ} and M_{YZ} while second PKF does it with assumed wrong values of M_{XZ} and M_{YZ}. Effects of wrong value of correlation matrices is demonstrated by evaluating normalized value of change in RMSE values of the filters. An increase in RMSE value means the filter is performing less accurately. Normalized RMSE is defined as follows

$$delta_{RMSE} = RMSE_M - RMSE \tag{35}$$

$$Percent_{delta} = delta_{RMSE} / RMSE * 100 \tag{36}$$

Where, $RMSE$ is Root Mean Square Error value of corresponding filter for correct M_{XZ} and M_{YZ} and $RMSE_M$ is Root Mean Square Error value of corresponding filter for assumed wrong values of above correlation matrices. For current example of 3 bus test system with PMUs placed on nodes 1 and 2, following analyses have been performed to evaluate performance of the developed PKF:

a. State Estimation (correct M_{xz} and M_{yz}) – from initial known values
b. Parameter estimation (correct M_{xz} and M_{yz}) – from initial known values
c. State Estimation (correct M_{xz} and M_{yz}) – from initial uncertain values
d. Parameter estimation (correct M_{xz} and M_{yz}) – from initial uncertain values
e. Effects of wrong M_{xz} and M_{yz} on RMSE of States
f. Effects of wrong M_{xz} and M_{yz} on RMSE of Parameters
g. Effects of wrong M_{xz} on RMSE of States
h. Effects of wrong M_{xz} on RMSE of Parameters
i. Effects of wrong M_{yz} on RMSE of States
j. Effects of wrong M_{yz} on RMSE of Parameters
k. Effects of correlation matrix values on estimated, V_{real}, V_{imag}, G, B and B_{sh} of the system

Effects of change in RMSE values have been demonstrated on bigger test systems i.e. IEEE 14, IEEE 30, IEEE 57 and IEEE 118 bus test systems with given PMU sets.

4.1 State Estimation using Developed PKF

Performance of developed PKF for state estimation on 3 bus test system when PMUs are placed at buses 1 and 2 is discussed here.

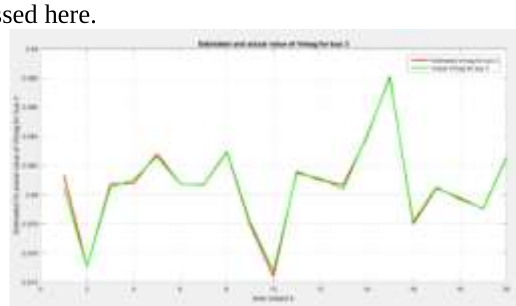

Fig. 3. Actual Vs Estimated value of Vimag at bus 3

In fig. 3 actual Vs estimated values of V_{imag} for bus 3 where PMU is not present has been shown for first 20 time instant of simulation.

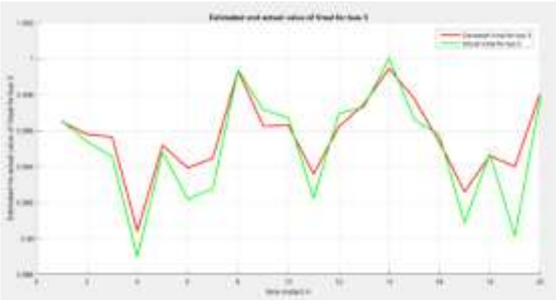

Fig. 4. Actual Vs Estimated value of V_{real} at bus 3

In fig. 4 actual Vs estimated value of V_{real} at bus 3 is given for first 20 instants of simulation. We can see that state estimator part of the developed PKF working correctly for present example.

4.2 Parameter Estimation using developed PKF

In this section we discuss performance of developed PKF on parameter estimation of 3 bus example network when PMUs are placed on nodes 1 and 2. Parameter estimation is performed by second Kalman filter in the PKF. Estimated Vs actual value of parameters of line 2-3 are discussed here.

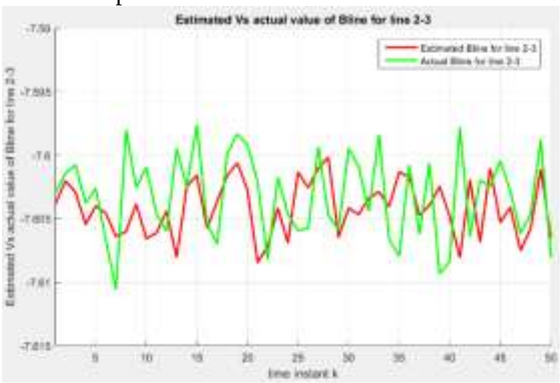

Fig. 5. Estimated Vs actual value of B_{line} for line 2-3

A comparison between estimated and actual value of susceptance of transmission line 2-3 is shown in fig. 5. We see that developed PKF estimates the parameter values within close range to actual value.

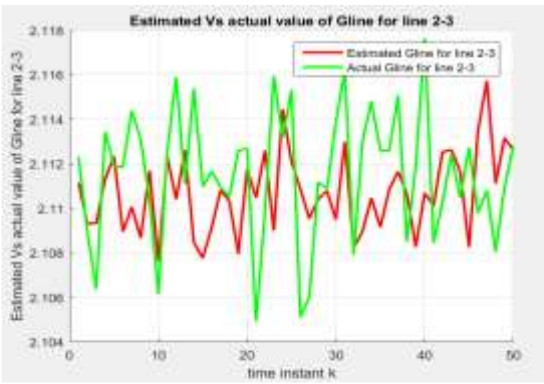

Fig. 6. Estimated Vs actual value of G_{line} for line 2-3

Figure 6 gives a comparison between actual and estimated values of conductance of transmission line 2-3 from the second filter from developed PKF. We see that estimated and actual value are reasonably close proving the capability of PKF to track parameter values of the network.

4.3 Estimation of states and parameters from initial uncertain value using developed PKF

In this section we discuss about the state/parameter estimation using developed PKF when initially some parameter values were uncertain. To test the estimation algorithm, we assume that initially values of admittance for line 2-3 was 2.6111 - j 7.1039. We know that these are not correct values for admittance of line 2-3 and it may affect the performance of dynamic state estimation in case of a regular Kalman filter.

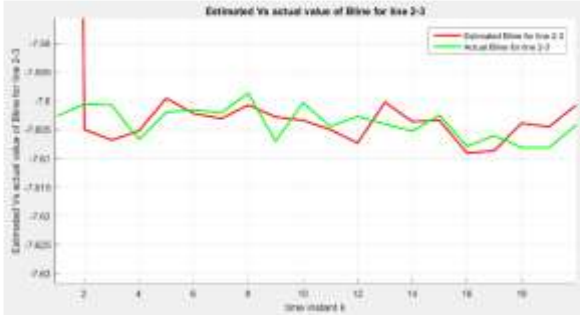

Fig.7. Estimated Vs Actual value of B_{line} from uncertain initial values

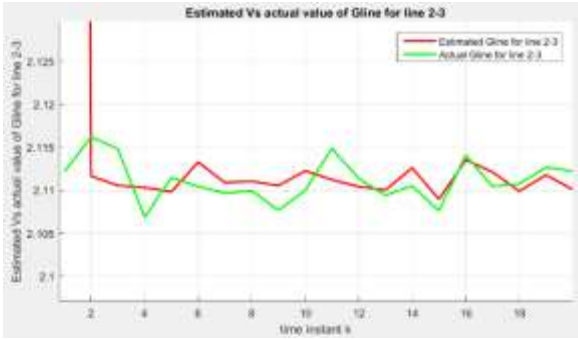

Fig.8. Estimated Vs Actual value of G_{line} from uncertain initial values

Figures (7-8) show estimated value of line susceptance and line conductance in case of uncertain values of line admittance initially available to the control center. We see that developed PKF can track values of line parameters within reasonably close range in this case as well.

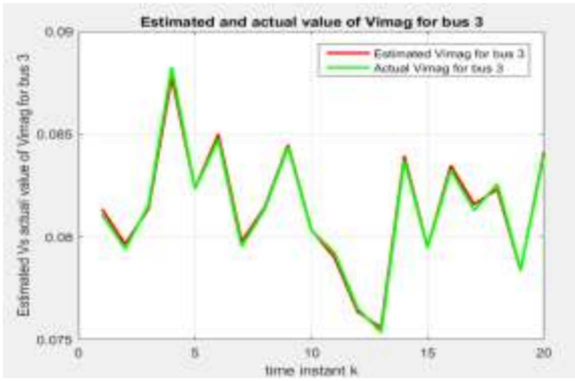

Fig.9. Estimated Vs Actual value of V_{imag} for bus 3 for uncertain initial value of parameters in line 2-3

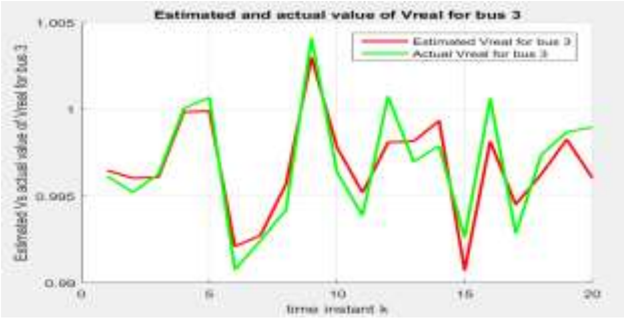

Fig. 10. Estimated Vs Actual values of Vreal for uncertain initial values of line parameters

Figures (9-10) give comparison between estimated and actual values of voltage phasors for bus 3 in case of initially uncertain values of line parameters in line 2-3 when PMUs are placed on nodes *1* and *2*. We see that even with uncertain values of line parameters available to us initially, the developed PKF is able to track system states as well as network parameters. Thus we see that both states and parameters of the power systems can be estimated successfully using developed algorithm.

In next section we see the effects of these correlation matrices on performance of state and parameter estimation of power systems. To analyze this performance two PKFs were implemented for each of the cases. First PKF uses correct values of correlation matrices for state and parameter estimation while second filter uses wrong values of correlation matrices M_{XZ} and M_{YZ}. The performance of PKFs for these two cases are compared using RMSE values for two cases.

4.4 State and Parameter Estimation with wrong values of M_{XZ} and M_{YZ}

In this section we discuss effects of various correlation matrices on states/parameter estimation of power systems network using synchrophasor measurements. For each of the examples, 500 Monte Carlo simulations were run to calculate RMSE values of the estimator. To keep the simulation process simple, values of M_{XZ} and M_{YZ} used to generate the process dynamics and observers were assumed equal to zero. First PKF was implemented with values of correlation equal to zero while for second PKF a random value of same dimension with elements within proximity of $Q_{X,k}$ and $Q_{Y,k}$ were generated for each of the cases as follows

$$M_{XZfilter} = 5e\text{-}7 * randn(size(M_{XZ})) \quad (37)$$

$$M_{YZfilter} = 5e\text{-}7 * randn(size(M_{YZ})) \quad (38)$$

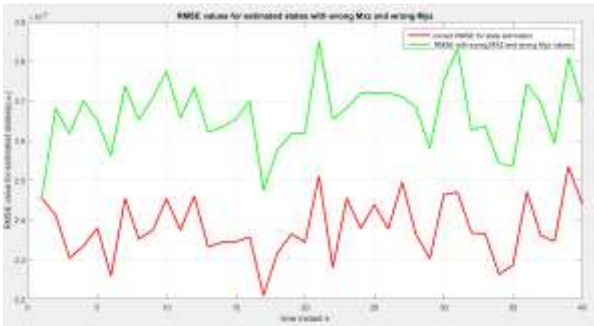

Fig. 11. Effect of wrong M_{XZ} and wrong M_{YZ} on performance of state estimator in PKF

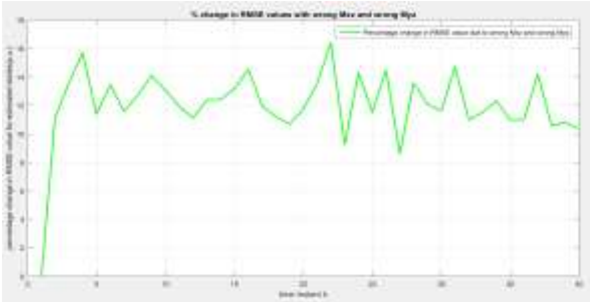

Fig. 12. Percentage change in RMSE of state estimator due to wrong M_{XZ} and wrong M_{YZ}

Figures (11-12) show effects of assuming wrong values of correlation matrices M_{XZ} and M_{YZ} for the state estimator by comparing the *RMSE* value of first filter (of PKF) for both cases. In fig 11 we clearly see that RMSE value increases indicating that the state estimator becomes less accurate when values of M_{XZ} and M_{YZ} are ignored in filter implementation. Figure 12 shows percentage increase in RMSE value of the state estimation filter due to using wrong value of mentioned correlation matrices. The parameter estimation process (second filter of the PKF) can be equally affected by using wrong correlation matrices.

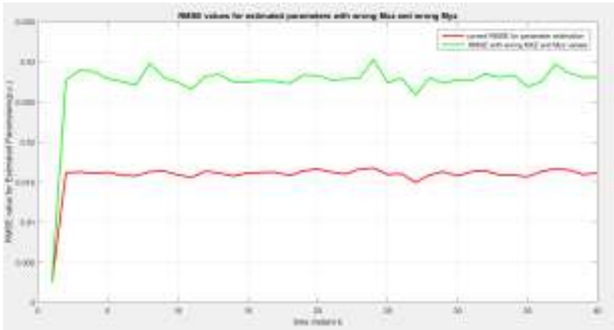

Fig. 13. Change in RMSE value of Parameter Estimator due to wrong M_{XZ} and wrong M_{YZ}

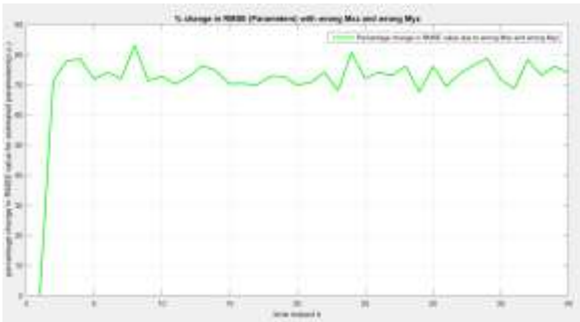

Fig. 14. Percentage change in RMSE in parameter estimation due to wrong M_{XZ} and wrong M_{YZ}

Figures (13 – 14) demonstrate effects of wrong values of correlation matrices M_{XZ} and M_{YZ} on RMSE of parameter estimation of the system. We see from fig. 13 that RMSE values of parameter estimator increases and stays high consistently by using wrong values of correlation matrices. Figure 14 shows percentage increase in RMSE values for this case for first 40 time instants of simulation.

This analysis can be further extended to parts of the state and parameter vectors as well. For example, here effects of wrong values of correlation matrices are evaluated only for estimated values of real parts of voltage phasors, represented as V_{real}.

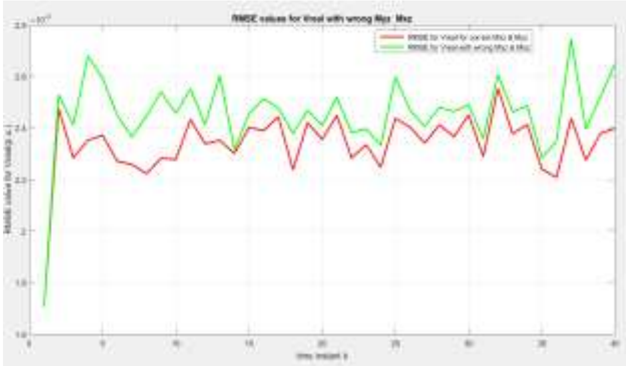

Fig. 15. Change in RMSE value for estimation of V_{real} for wrong correlation matrices

Figure 15 elaborates effect on RMSE for state estimator for section estimating real part of voltage phasors of the state vector. This analysis can give further insight into estimation process and effects of various factors on estimated values of different parts of the state/parameter vectors. Similar analysis can be done for V_{imag}, b_{line}, g_{line} and b_{sh} vectors as well.

4.5 RMSE values for wrong M_{XZ} and correct M_{YZ}

The effects of wrong values of M_{XZ} on state and parameter estimation of power systems are shown with the help of developed PKF in this section. To further verify performance of developed PKF, we hypothesize a scenario with correct information of M_{YZ} and wrong information of M_{XZ} used in implementation for state and parameter estimation of power systems. Both process and observer model were generated with zero values of M_{XZ} and M_{YZ} while when implementing the filter one PKF was implemented with both values correct while another PKF was implemented using wrong value of M_{XZ}. Wrong value of M_{XZ} was generated using (37). The effects of these correlation matrices on performance of state/parameter estimation are discussed here.

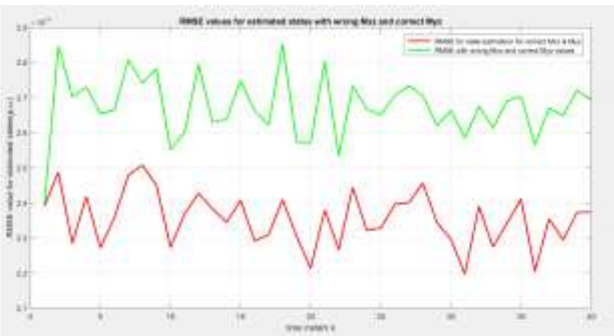

Fig. 16. Effect of wrong correlation matrix M_{XZ} on state estimator

In fig. 16 we see that wrong value of correlation matrix M_{XZ} deteriorates the filter performance as the RMSE value of state estimator increases indicating the filter becoming less accurate.

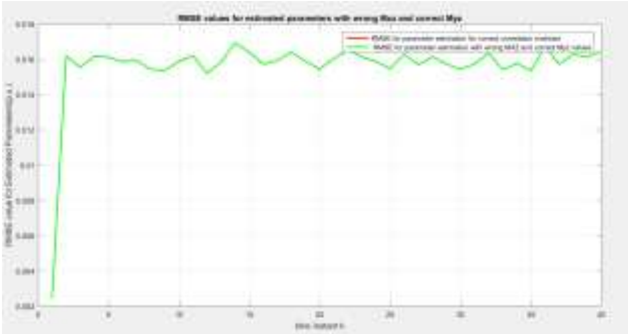

Fig. 17. Effect of wrong correlation matrix M_{XZ} on parameter estimation

In fig. 17, it is observed that having a wrong value of correlation matrix M_{XZ} does not affect the performance of parameter estimator as expected. It can be conjectured from the dynamics of system states and network parameters that they are independent of each other. A similar effect is found on performance of state estimator when wrong values of correlation matrix M_{YZ} was considered for filter implementation, meaning that wrong values of M_{YZ} only deteriorates performance of parameter estimation process while state estimation remains immune to it.

4.6 State and parameter estimation for IEEE 14 bus test systems

In this section effects of correlation matrices on state/parameter estimation of IEEE 14 bus test system is discussed when PMUs placed for measurements consist of minimum set of PMUs required for complete observability. For this example, PMU set following this criteria would be placement at buses 2, 6, 7 and 9. The set of correlation matrices for the implementation of a second PKF as described before are generated using (37-38) as before.

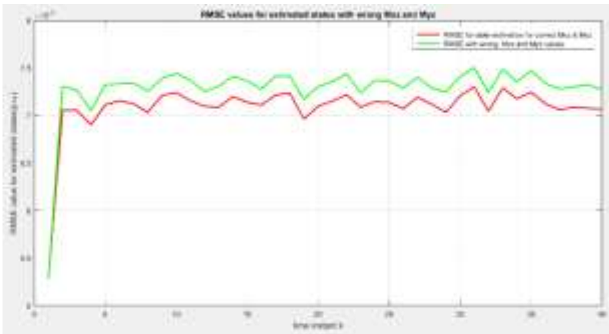

Fig. 18. Effect of wrong M_{XZ} and M_{YZ} on RMSE of state estimator for IEEE 14 bus test system

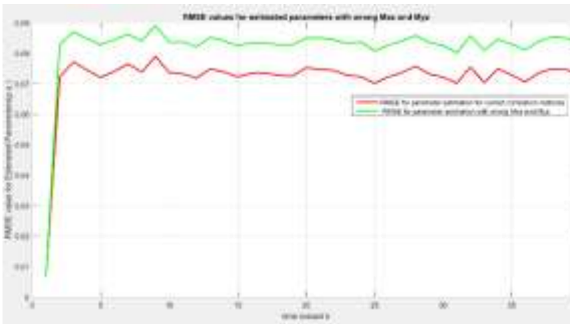

Fig. 19. Effect of wrong M_{XZ} and M_{YZ} on RMSE of parameter estimation for IEEE 14 bus test system

Figures 18-19 show effects of wrong correlation matrices on RMSE values of state as well as parameter estimation of power systems for IEEE 14 bus test system. It is evident that RMSE value increases for both filters, making them to be less accurate as values of correlation matrices M_{XZ} and M_{YZ} are not appropriately included in filter implementation.

To further analyze the performance effects of correlation matrices M_{XZ} and M_{YZ} is shown on state/parameter estimation using developed PKF for available redundancy in measurement. For this case additional PMUs were placed on the network and developed PKF was implemented. A new placement set for this example is PMUs at nodes 1, 2, 3, 6, 7, 9, 12 and 14. The correlation matrices were generated using (37-38).

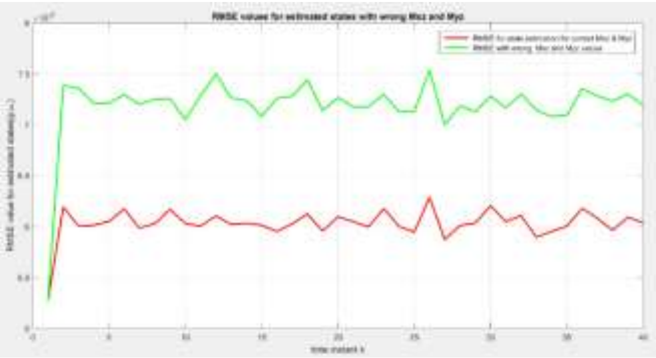

Fig. 20. RMSE for state estimation for IEEE 14 bus test system for new PMU set

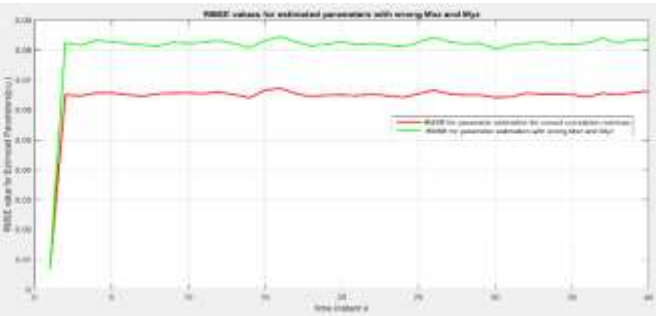

Fig. 21. RMSE for parameter estimation for IEEE 14 bus test system for new PMU set

Figures (20-21) shows change in RMSE values of state and parameter estimation using developed PKF when a larger set of PMUs with redundancy in measurement is placed on the network.

It should be evident that developed PKF can do both state and parameter estimation but to keep the analyses simple we are only showing performance of state estimator for following section. For following section, effects of wrong correlation matrices on state/parameter estimation of IEEE 30, IEEE 57 and IEEE 118 bus test system has been shown for the case where minimum number of PMUs were placed to make the network completely observable.

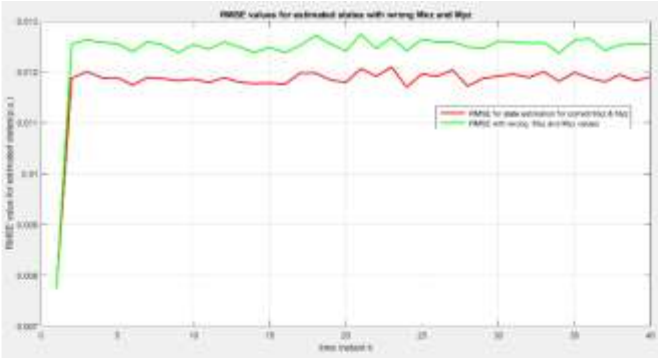

Fig. 22. Effect of M_{XZ} and M_{YZ} on RMSE of state estimator for IEEE 30 bus test system

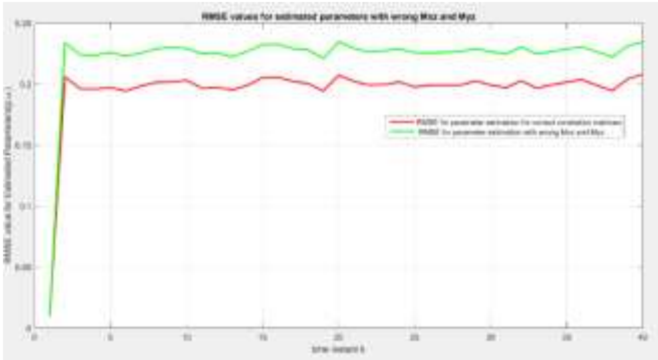

Fig. 23. Effect of M_{XZ} and M_{YZ} on RMSE of parameter estimator for IEEE 30 bus test system

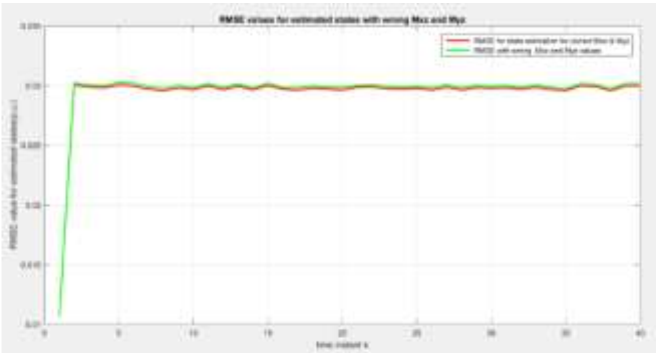

Fig. 24. Effects of M_{XZ} and M_{YZ} on state estimation for IEEE 57 bus test system

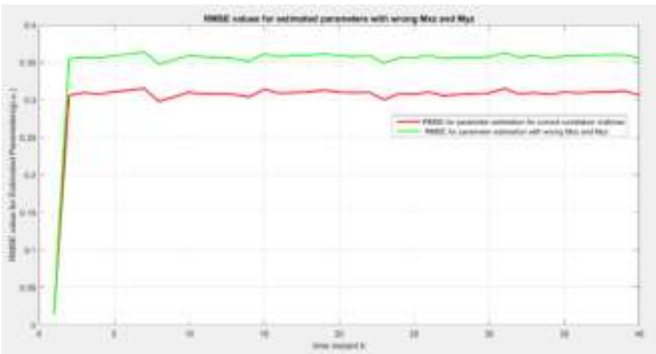

Fig. 25. Effects of M_{XZ} and M_{YZ} on parameter estimation for IEEE 57 bus test system

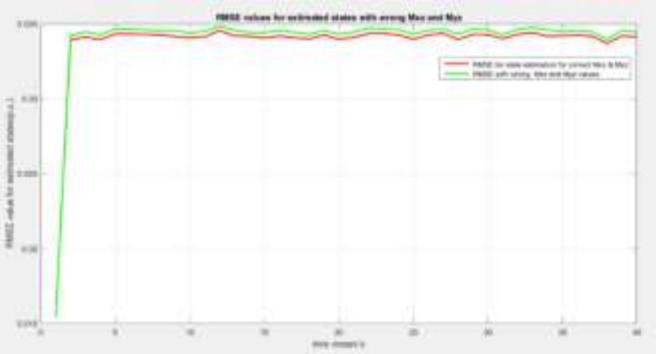

Fig. 26. Effects of M_{XZ} and M_{YZ} on RMSE of state estimator for IEEE 118 bus test system

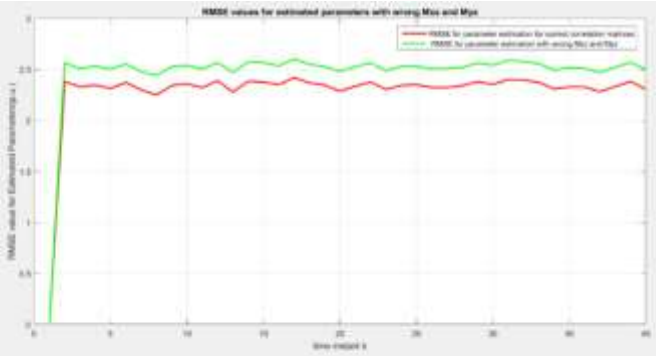

Fig. 27. RMSE for parameter estimation for IEEE 118 bus test system

Figures (22-27) show effects of correlation matrices on state and parameter estimation of power systems for various example bus networks. As we see in every case, the RMSE values of the filters increase when using wrong values of correlation matrices for filter implementation, proving them to be less accurate. Thus the developed algorithm performs better for all these examples.

5. Conclusion

State estimation of power systems for various cases have been done in literature before. Recently people have focused on estimating parameters of power systems network using various methods. This paper develops a novel method to implement state estimation using Parallel Kalman Filter for bilinear model systems when there are correlations present among noise in dynamics of each partitions of state vectors with measurement noise. State estimation for a bilinear system model in presence of correlation in noise of dynamics between various parts of system states has never been done before. This theory was found of direct use in case of changing parameter values of the power system networks. For this case, a method to do both state and parameter estimation of power systems using synchrophasor measurements was required. The developed PKF was implemented to do state and parameter estimation of power systems in presence of synchrophasor measurements. Developed algorithm was able to do state estimation for various cases as well as parameter estimation when initial values of parameters known were uncertain. Monte Carlo simulations were performed to evaluate performance of state and parameter estimation operations on the example networks. For each of the example networks, different measurement sets were generated for various PMU placement sets and state and parameter estimation was performed using developed PKF. The effects of correlation matrices were clearly shown on the performance of both state and parameter estimation of power systems by using RMSE values for the implemented filters. Developed algorithm was implemented on IEEE 14, IEE 30, IEEE 57 and IEEE 118 bus test systems for different measurement sets and results were found as expected. In addition to this, developed PKF can be applied to state estimation of power systems for regular cases of fixed parameters or cases where zero correlation is present between noises in state dynamics and dynamics of the parameters.

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Appendix A

Here we discuss development of Parallel Kalman Filter for bilinear systems when correlation between noise in dynamics in state vectors and noise in measurements are present. In [27] authors have introduced PKF for bilinear system model without presence of any correlation between process and measurement noises. Let the system state vector can be partitioned into two vectors Z and θ. Variable Y_k represents measurements of the system at time instant k, which depends on both states and parameters as a bilinear system.

Let the bilinear discrete time representation of the dynamic system is given

$$\begin{pmatrix} Z_{k+1} \\ \theta_{k+1} \end{pmatrix} = \begin{pmatrix} A_{11}+F_1(\theta_k) & A_{12} \\ A_{21} & A_{22}+F_2(Z_k) \end{pmatrix} \begin{pmatrix} Z_k \\ \theta_k \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} u_k + \begin{pmatrix} \varepsilon_{k1} \\ \varepsilon_{k2} \end{pmatrix} \quad (39)$$

Random variables ε_{k1} and ε_{k2} represent zero mean white noise with known covariance matrices Q_{k1} and Q_{k2} respectively. Also,

$$F_1(\theta_k) = \sum_{i=1}^{q} F_{1i}\theta_{ki} \quad (40)$$

$$F_2(Z_k) = \sum_{i=1}^{q} F_{2i}Z_{ki} \quad (41)$$

Measurement equations for bilinear model is given by

$$\frac{Y_k}{Y_k} = \frac{H_1 Z_k + H_2 \theta_k + C_1(\theta_k) Z_k + V_k}{H_1 Z_k + H_2 \theta_k + C_2(Z_k) \theta_k + V_k} \quad (42)$$

It is assumed that the correlation between Z and Y as well as correlation between θ and Y is known and is denoted by M_{ZY} and $M_{\theta Y}$ respectively. Also, V_k represents measurement noise vector which is a white noise with zero mean and known variance matrix R_k.

Fundamental idea behind PKF is derived from game theory where each of the opponent makes their decision based as an optimal response to the decision made by the other opponent. In context of state estimation theory, the move decided by each of the player can be considered as output of the filter where objective is to minimize the covariance of estimation errors. Filters are implemented recursively with arrival of each of the measurements. Thus output of the filters can be obtained by minimizing following objective functions

$$\min_{\hat{X}_{k|k}} J_1(\hat{X}_{k|k}, \hat{Y}_{k|k}^{*}) \quad (43)$$

$$\min_{\hat{Y}_{k|k}} J_2(\hat{X}_{k|k}^{*}, \hat{Y}_{k|k}) \quad (44)$$

Where, J_1 and J_2 are covariance of estimation error for each of the filters. The superscripts * stands for optimal solution and ^ stands for estimated value of the states. Subscript i/j denotes estimated value of the variable at time instant i based on information up to time instant j. to be able to implement two Kalman filters parallel to each other, it is imperative that instead of using optimal value (output of the other filter), predicted optimal value (predicted output of the other filter) is used as parameters for the implementation of first filter. This means that instead of $\hat{Y}_{k|k}^{*}$, variable $\hat{Y}_{k|k-1}^{*}$ will have to be used for first Kalman filter and vice versa. Thus each of the system becomes a linear subsystem which has time varying parameters.

Prediction:

Prediction is same as a standard Kalman Filter for this model and is given by

$$\hat{Z}_{k+1|k} = [A_{11} + F_1(\hat{\theta}_{k|k-1})]\hat{Z}_{k|k} + A_{12}\hat{\theta}_{k|k-1} + B_1 u_k \quad (45)$$

$$\hat{\theta}_{k+1|k} = [A_{22} + F_2(\hat{Z}_{k|k-1})]\hat{\theta}_{k|k} + A_{21}\hat{Z}_{k|k-1} + B_2 u_k \quad (46)$$

Predictions of covariance matrices of estimation errors for first filter can be given by

$$P_{k|k-1}^{(1)} = [A_{11} + F_1(\hat{\theta}_{k|k-1})] \; P_{k-1|k-1}^{(1)} \, [A_{11} + F_1(\hat{\theta}_{k|k-1})]^T + Q_{k1} \quad (47)$$

Similarly, prediction for covariance of estimation error for second Kalman filter is given by

$$P_{k|k-1}^{(2)} = [A_{22} + F_2(\hat{Z}_{k|k-1})] \; P_{k-1|k-1}^{(2)} \, [A_{22} + F_2(\hat{Z}_{k|k-1})]^T + Q_{k2} \quad (48)$$

Update:

Here we will discuss derivation of first filter out of two interlaced filters. Derivation of second Kalman filter Update process has steps similar to first Kalman Filter.

The predicted measurement at time instant k for first filter is given by

$$\Delta Y_k = Y_k - \hat{Y}_{k|k-1} \quad (49)$$

The covariance of innovation for first filter

$$S_k = cov(\Delta Y_k) \quad (50)$$

The covariance of innovation error for first filter, which will have presence of M_{ZY}, is given by

$$S_k^{(1)} = \left(H_1 + C_1(\hat{\theta}_{k|k-1})\right)^T P_{k|k-1}^{(1)} \left(H_1 + C_1(\hat{\theta}_{k|k-1})\right) + \left(H_1 + C_1(\hat{\theta}_{k|k-1})\right) M_{ZY} + M_{ZY}^{\;T} \left(H_1 + C_1(\hat{\theta}_{k|k-1})\right)^T \quad (51)$$

Equation for Kalman gain for first filter is given by (based on expression for standard Kalman filter with presence of correlation between process and measurement noise)

$$K_k^{(1)} = [P_{k|k-1}^{(1)} \left(H_1 + C_1(\hat{\theta}_{k|k-1})\right) + M_{ZY}][S_k^{(1)}]^{-1} \quad (52)$$

Updated estimate for first filter is given by

$$\hat{Z}_{k|k} = \hat{Z}_{k|k-1} + K_k^{(1)} \left[Y_k - \left(H_1 \hat{Z}_{k|k-1} + H_2 \hat{\theta}_{k|k-1} + C_1(\hat{\theta}_{k|k-1})\right) \hat{Z}_{k|k-1}\right] \quad (53)$$

Expression for update of covariance of estimation error is given by

$$P_{k|k}^{(1)} = P_{k|k-1}^{(1)} - K_k^{(1)} \, [H_1 + C_1(\hat{\theta}_{k|k-1})]^T P_{k|k-1}^{(1)} \quad (54)$$