

Modal Analysis of a Small H-Darrieus Wind Turbine Based on 3D CAD, FEA

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Abstract-Rotary machines have many rotating structures necessity design-optimization. Their structure motions are controlled at low-frequency by rigidity, at high-frequency by inertia and at resonance level by damping. Using modal model, dynamic design of the structure developed can be predicted, analyzed and improved. Recently, H-Darrieus wind turbine (HDWT) has received considerable attention due to its inherent structural characteristics. This machine intends a promising design of renewable energy conversion system in urban and isolated areas. Though, the system suffers from several dynamic problems caused by geometry, centrifugal and aerodynamic cyclic loadings. Present paper investigated dynamic design-optimization of a three-bladed (HDWT) based on its natural structural parameters using 3D-CAD-FEA using SolidWorks modeling and simulation software. From simulation results obtained, (i) the minimum static safety factor of the wind turbine materials is equal to 1.4. It is greater than that recommended by the international standard IEC61400-1, assessing an acceptable value to 1.35 when the mass of the system is not obtained by weighting; (ii) the first three natural frequencies of the system are (17.73, 17.99 and 21.07Hz), the resultant mass participates (10.55, 10.44 and 0.04%), the modal damping (9.19, 9.17 and 9.05%), also resonant amplification (5.44, 5.44 and 5.52), magnitude ratios (100, 97.13 and 70.82%) are calculated and mode shapes associated are predicted and analyzed; and (iii) critical operating conditions of wind turbine under forced excitations due to the wind speeds at various regimes are also treated and assessed. The static and dynamic stability and reliability of the system are shown since all quality indicators tested are consistent according to structure dynamics standards made in steel materials.

KeywordsH-Darrieus wind turbine, Structural dynamics, Design-optimization, Static-analysis, Modal-analysis, 3D-CAD-FEA.

1. Introduction

Nowadays, rotary machines are used in almost overall aspect of our life ranging from automobiles, aeroplanes, vacuum cleaners, wind turbines, and power plants. All machines have many rotating structures whose dynamics need to be designed, analysed and enhanced. These systems have become increasingly extra complex, faster, flexible and yet strong in order to increase their output, shortening development process, reducing weights and costs. Consequently, they develop into an extremely prone to vibration and noise due to the particular dynamic loadings which cause variation in stress and strain. These forces excite components or even the entire machine to vibrate dangerously during operating machinery [1]. Resonant

vibration is mainly produced by interactions between the inertia forces and elastic forces characterized by geometry and material properties of the system under structure deformation. Forecasting structural parameters and dynamics behaviour constitute the most important stage in design process of the system. To better describe, understand and avoid any structural vibration problem, resonant frequencies, mode shapes and structural damping of the structure require to be identified and quantified [2]. Modal analysis has become a standard tool for structural dynamics problem analysis and design-optimization in research and industry. It can be helpful in estimating the structural parameters, energetic performances, predicting fatigue stressing and other issues of the system [3]. Recently, the vertical axis straight blades H-Darrieus wind turbine (HDWT) has

received great attention due to its intrinsic characteristics not actively controlled [4]. The (HDWT) is Omni-directional, less noise, and had its entire heavy component sited close to the ground level, making the maintenance of the (WT) quite easy. Blades are maintained at several positions and aerodynamically efficient than other (WT) types which promise to be more effective cost. The blade pitch design can be varied, so angle of attack gave high lift to drag ratios can be maintained for longer time during a period of the machine. The physics flow is expected to be complex due to the wake with structures flow of different length scales, high directional variability, large skew angles and increased turbulence intensity. For the same overall sizes the (HDWT) has a swept area about 50% greater than the curved Troposkine-shaped blade design. Mechanically the Darrieus wind turbines better resist to higher wind speeds and their output when putted on the urban rooftop building have revitalized the interest in the system. Moreover, it is relatively easy to manufacture straight blades (WT) than curved blades of the appropriate forms. The dynamics analysis of the three-bladed of (HDWT) is fairly complicated. The system operates at low Reynolds numbers; its blades are highly prone to separation which produces fluctuation in the torque, induces vibrations and dynamic stalling [5]. The stability, reliability and dynamic response level of the machine forecasted by analytical modelling are usually not satisfactory until validated by experimental way. The identification process of dynamic structural parameters is more difficult in practice, due to the presence of identical pairs of frequencies related to the axisymmetric geometry of the (WT) and therefore special testing are required to accurate extraction of modes. The system contains a drive train device (low and high speed shaft, gear box, generator, and break disc), do not causes high bending strains in the blades or cross-arms, since they are stiff in this direction by the nature of their aerofoil section but, they have time varying factors in their motion equation and thus lead to behaviours untreated rigorously using existing experimental modal analysis methods [6]. The (HDWT) is subjected to the blade bending increasing from centrifugal effects, varying of torque and thrust three times per revolution. The radial aerodynamic forces combined with the high centrifugal loading, provides a bending strain variation in the rotor shaft and mast. The blades perform relatively a constant wind shear, though the critical cyclic stress of the cross-arms and blades are super-imposed on an increasingly large static gravity stress; this will harmfully touch the fatigue performance of the system [7].

The present paper investigation on the dynamic design of a straight three-bladed (HDWT), built to achieve high stability, safety, reliability and long lifetime. It is demanded to provide economic power based on static and modal analysis approaches to predict displacements, security factor, resonant parameters (natural frequencies, mode shapes, and damping factors), using 3D CAD, FEA industrial modelling and simulation software under SolidWorks code to predict and to avoid resonance to happen in the (WT) structure due to wind speeds. The excited frequencies are calculated and the resonant conditions of the (WT) are fixed and analysed. Finally, the design quality of the system is appreciated based

on static security factor, amplification factor and severity of normalized amplitudes of vibration modes agreed by structural dynamics standards IEC [8].

2. Modeling and Simulation Procedure

2.1. Solid Modeling

The FEA process of flexible structures was generally classified into three major basic steps: Pre-process, Process and Post-process [8]. The first step is divided into geometric and numerical modelling. Modelling tasks include de-featuring and simplifying “Fig.1a”. De-featuring involves eradicating the features that are not affecting parameters of interest significantly to improve the FEA processing time. Simplifying involves abridging or even changing the geometry of the 3D solid model. Also, important mathematical modelling tasks include performing the suitable analysis kind based on FEA purpose static, dynamic, fatigue and other, implementing correct fixtures, assigning materials, applying needed loading and selecting the proper types of meshing elements “Fig.1b”. The turbine is built in alloy Aluminium, the chord profile of blades is NACA0018, the gearing is in treated steel AISI1020 and the generator in carbon steel. The second step includes system meshing “Fig.1c”, by transforming the solid model into discrete finite elements and simultaneously solving algebraic equations of the system motion to determine interest mechanical and structural parameters. The (WT) was meshed in 675,801 elements linked to 218,668 nodes using tetrahedral elements to generate 661,182 DoF. Finally, the third step consists in results appreciation to guarantee that the FEA model represents numerical model, and characterizes adequately the physical system.

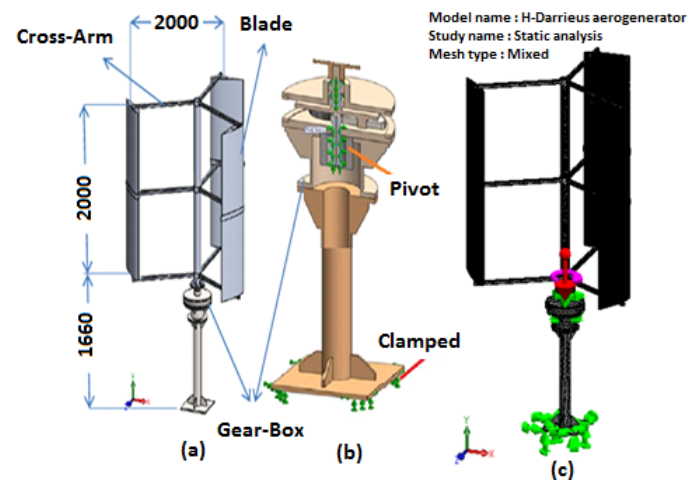


Fig. 1. Design procedure of the wind turbine: a. 3D solid model; b. boundary conditions and loading; c. Finite element meshing.

2.2. Static Analysis

Before modal analysis, the strength (yield stress) σ_e of the (HDWT) should be verified by performing static analysis of the (WT) structure subjected to the competition between critical centrifugal force due to the rotation of 30rd/s applied

statically and elastic forces caused by material proprieties [10]. From eigenvalues of stresses $\sigma_X, \sigma_Y, \sigma_Z$ in X, Y and Z directions of the constraint tensor, and based on rupture theory of ductile materials, the Von-Mises resistance criterion of maximal material stress in principal coordinates is expressed as the form:

$$\sigma_{VM} = \sqrt{0.5[(\sigma_X - \sigma_Y)^2 + (\sigma_Y - \sigma_Z)^2 + (\sigma_Z - \sigma_X)^2]} = \sigma_e / S_f(1)$$

S_f is the safety factor of system materials. Static analysis results using 3D-CAD-FEA modelling and simulation, including mechanical parameters predicted of the system design are shown in Fig. 2.

2.3. Modal Analysis

Modal analysis theory in 3D CAD, FEA consists to use natural coordinates to replace nodal coordinates by coordinate's transformation. To decouple motion equations, the physic coordinates are replaced by modal coordinates. The linear analysis is concerned with the solution of the differential equation as follows [11]:

$$[M]\{\ddot{U}\} + [D]\{\dot{U}\} + [K]\{U\} = \{F(t)\}(2)$$

Where $[M], [D]$ and $[K]$ are resp. mass, damping and stiffness matrices, real, symmetric and positive define. $\{U\}, \{\dot{U}\}, \{\ddot{U}\}$ are resp. displacement, speed and acceleration time-dependent vectors of the system, and $\{F(t)\}$ external force vector. This force consists of harmonic aerodynamic forcing function independent of structural response. It does not affect the stability of the (WT) and it is therefore neglected. By considering undamped structure, the homogenous differential Eq. (2) is then solved by introducing harmonic solution with the same pulsation $\{U(t) = \{\Psi\} \exp(j\Omega t)\}$, ($j^2 = -1$) to find eigenvalues and eigenvectors.

$$([K] - \Omega^2[M])\{\Psi\} \exp(j\Omega t) = \{0\} \quad (3)$$

$[\Omega^2], [\Psi]$ are square spectral matrices of natural frequencies and corresponding mode shapes. Natural modes are normalized according to inertia or stiffness matrix using orthogonally of modes:

$$\Omega_r^2 = k_r / m_r = \{\Psi_r\}^T [K] \{\Psi_r\} / \{\Psi_r\}^T [M] \{\Psi_r\}, \quad r = 1, 2 \dots n(4)$$

m_r, k_r are resp. modal mass and modal stiffness of the system at r^{th} mode. Natural modes are normalized by considering $\{\phi_r\} = \{\Psi_r\} / \sqrt{m_r}$. The projection of the system of Eq. (2) on modal bases provides in generalized coordinates q_r :

$$\ddot{q}_r(t) + \Omega_r^2 q_r(t) = \{0\}(5)$$

The orthogonally of modes expressed only the mechanical characteristics of the structure, inertia force or stiffness force developed into a particular mode and do not participates into the other modes. Thus, the free motion of the system is the superposition of all natural vibration modes:

$$U(x, t) = \sum_{r=1}^n \Psi_r(x) q_r(t) \quad (6)$$

Under viscous damping assumption, we use typically in practice the linear Rayleigh form $[D] = \alpha[M] + \beta[K]$, where α and β are mass and stiffness contributions in the system structure. Using orthogonally proprieties of modes and normalizing the mass matrix by unity give:

$$d_r = \{\phi_r\}^T [D] \{\phi_r\} = \alpha + \beta \Omega_r^2 = 2\xi_r \Omega_r \quad (7)$$

Modal damping d_r in Eq. (7) is specified from two successive frequencies Ω_r and Ω_k to solve for α and β .

$$\alpha = 2\Omega_r \Omega_k (\Omega_r \xi_k - \Omega_k \xi_r) / (\Omega_r^2 - \Omega_k^2); \quad \beta = 2(\Omega_r \xi_k - \Omega_k \xi_r) / (\Omega_r^2 - \Omega_k^2) \quad (8)$$

In modal coordinate, the damped system motion without external force of Eq. (2) becomes:

$$\ddot{q}_r(t) + 2\xi_r \Omega_r \dot{q}_r + \Omega_r^2 q_r(t) = \{0\} \quad (9)$$

In proportional damping case, there is no coupling between vibration modes of the system. The complex natural frequency of the r^{th} mode may be expressed by

$$\Omega_r = -\Omega_r \xi_r \pm j \Omega_r \sqrt{1 - \xi_r^2}, \quad j^2 = -1 \quad (10)$$

Here the damped natural term is $\Omega_d = \Omega_r \sqrt{1 - \xi_r^2}$. It is evident that for small damping factors, the natural vibration modes are real, and they are not affected by damping $\Omega_d = \Omega_r$. Moreover, the effect on the transient of the terms ξ_r and Ω_r can readily be seen. The larger the product $\xi_r \Omega_r$, the faster the transient will decay. These terms also affect the damped natural frequency of oscillation of the transient Ω_d , which varies directly as the undamped natural frequency and decreases with an increase in damping ratio.

3. Dynamic Design Assessment

3.1. Mass Participate

This indicator included n vibration modes and defined as the mass participate (MP) divided by the total mass in that direction without considering stiffness based on a unit base acceleration in a particular principal directions (X, Y, Z), and calculate the base shear due to that load [12]:

$$X_{mass,i} = \sum_{r=1}^n (\ddot{q}_r X_i)^2 / \sum m_{X_i}, \quad X_i = X, Y, Z \quad (10)$$

3.2. Amplification Factor

This indicator measures the ratio of total vibration energy of the structure to the total vibration energy lost due to the viscous damping, hysteresis in material, dry friction into joints, internal interactions, clamped connections and acoustic wave propagation lost during vibration cycle. For light damping structures this factor has typically values from 5 to 50, depending on the nature of materials and fixtures. For structures built in metal, the indicator for r^{th} vibration mode is given in [13].

$$Q_r = 1 / \xi_r \sqrt{1 - \xi_r^2} \cong 1 / 2\xi_r \quad (11)$$

Here the damping factor is approached by expression $\xi_r = 1 / (10 + 0.05\Omega_r)$ in [14].

3.3. Resonant Amplitude Ratios

Since the generalized mass, force, and damping for all resonance modes are the same, the following relationship exists between maximum responses and natural frequencies of the structure.

$$|A_r|_{max}/|A_1|_{max} = \Omega_1^2/\Omega_r^2 \quad (12)$$

3.4. Forcing Frequencies

The excited frequencies of the (WT) Ω_f (Hz) caused by wind speeds V_w (m/s) can be expressed as a function of design Tip Speed Ratio of design $\lambda_d = 3$, turbine radius R_t (m) and blade number N_b by expression:

$$\Omega_f = N_b \lambda_d V_w / 2\pi R_t \quad (13)$$

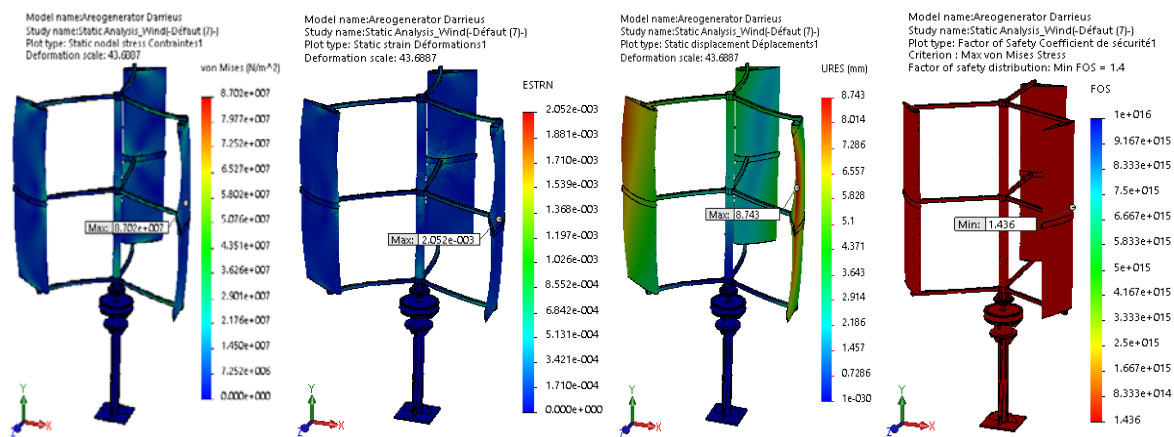
4. Results and Analysis

From 3D CAD, FEA modelling and simulation results of the present study, the following conclusions are drawn:

- The critical value of stress is 87MPa, strain is 0.002 and displacement is 8.74mm. Therefore, the static local and global stability of the (WT) structure is confirmed since the minimal security factor of materials obtained is equal to 1.4. This value is greater than the value recommended by the international standard IEC61400-1, which fixes the safety factor at 1.35 if the mass of the wind turbine is not given by weighting [8].
- From "Fig.3", due to the high similarity between two longitudinal symmetries of the (WT) in perpendicular directions X and Z, the first two bending modes are yielding to the close natural frequencies 17.731Hz, 17.991Hz. These vibration modes have a good agreement with the fundamental vibration theory of Euler-Bernoulli beam with lumped masses in bending vibration modes.
- During the symmetric response (Fig.3a, b, d and e), the (WT) performed bending and slightly coupling (bending-torsional) modes in X and Z directions, in which, the mass participate is one of the most important parameter

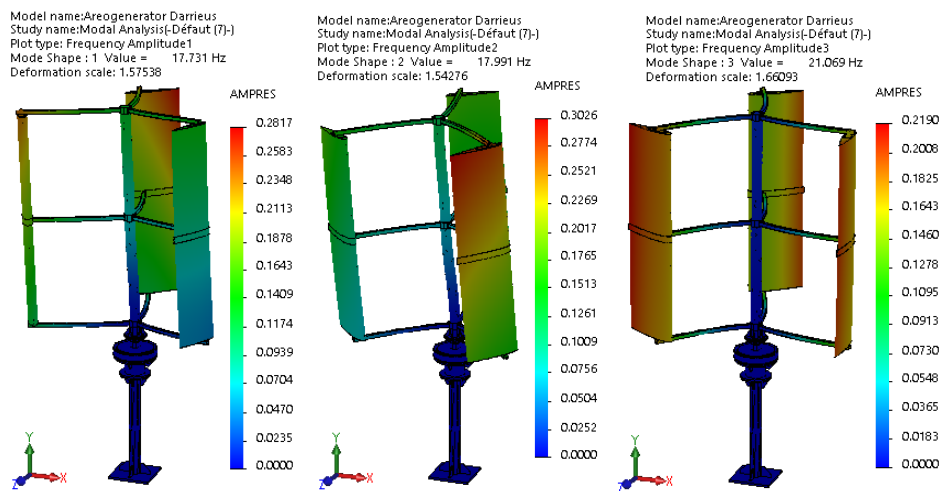
of the system dynamic design "Table 1.". However, during the anti-symmetric response "Fig.3c", the system executes torsional vibration modes. In this state, the inertia of the system is the most important parameter of the system dynamic design. The first and second bending modes in X and Z directions are characterized by a most resultant important (MP) less than 8.33%.

- Based on Table 1 and Eq. (4), the (WT) in torsional vibration mode has the weakest values of rigidities equals to 2.23N/m, 0, and 4.84N/m in X, Y, and Z directions respectively. Therefore, the system structure should be firstly endorsed in torsion if it is necessary.
- The Table 1 shows that the bending vibration modes are characteristics of wind speed around 111.4m/s and 113m/s, the torsional modes correspond to 132.38m/s and for the coupling mode correspond to 179.59m/s and 183.46m/s respectively.
- In the "Table 1.", when natural frequency increases, the amplification factor increases gradually until it passes by a maximum in the vicinity of $\Omega_f = \Omega_r$, inducing the resonance of the system. This maximum value is estimated by Eq. (11). This resonance is therefore directly determined by the damping; lightly damped structures typically have an amplification factor between 5 and 50, depending of materials and joints [13].
- The Table 1 showed also that the specific resonant amplitudes decrease speedily with the augmentation of natural frequencies, which showed dynamics stability and reliability of the (WT) according to the international standards of structure dynamics.
- The Table 2 showed that the (HDWT) has many natural vibration frequencies bounded between 18 and 31Hz. They appeared in the structure of (WT) at feeble wind regime for (TSR=2) where V_w is up to 19m/s, also at nominal operating regime (TSR=3) where V_w is up to 13m/s and at fort regime (TSR=4) when V_w is greater than 10m/s. If we limited the output wind power at $V_w=10$ m/s, the resonance do not happen in the structure of the wind turbine. However if we decide that the (WT) works at wind speed greater than $V_w=10$ m/s, an adequate control device is necessary to limit dynamics stresses and deformations of the system.

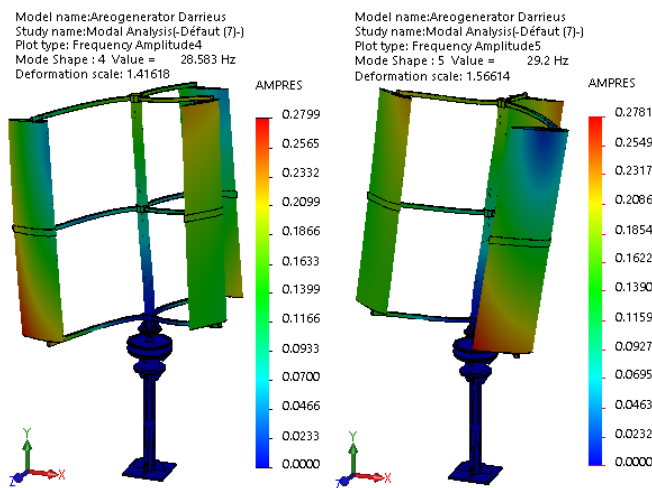


a. Stresses b. Strains c. Displacements d. Security factors

Fig. 2. Static analysis of the (HDWT)



a. X Bending: $\Omega_1 = 17.731 \text{ Hz}$. Z bending: $\Omega_2 = 17.991 \text{ Hz}$. Torsional: $\Omega_3 = 21.069 \text{ Hz}$



d. Coupling: $\Omega_4 = 28.583 \text{ Hz}$. Coupling: $\Omega_5 = 29.2 \text{ Hz}$

Fig. 3. First five natural vibration modes of the wind turbine

Table 1. Structural parameters and quality factors of the wind turbine

Mode no.	Ω_r (Hz)	$V_r = \Omega_r R_t$ (m/s)	PMR (%)			ξ_r (%)	Q_r (–)	$ A_r _m / A_1 _m$ (%)
			X	Y	Z			
1	17.731	111.4072	8.3275	0.0001	6.4767	9.19	5.4433	100.0000
2	17.991	113.0408	6.3011	0.0009	8.3277	9.17	5.4498	97.1306
3	21.069	132.3804	0.0169	0.0000	0.0366	9.05	5.5267	70.8237
4	28.583	179.5923	0.2399	0.0000	0.1064	8.75	5.7146	38.4814
5	29.200	183.4690	0.2585	0.0261	0.3023	8.73	5.7300	36.8723

Table 2.Forcing frequencies in Hertz of the wind turbine at different wind regimes

$\lambda_d V_w$ (m/s)	0.1	0.5	0.8	1	2	3	4
1	0.0477	0.2387	0.3820	0.4775	0.9549	1.4324	1.9099
4	0.1910	0.9549	1.5279	1.9099	3.8197	5.7296	7.6394
7	0.3342	1.6711	2.6738	3.3423	6.6845	10.0268	13.3690
10	0.4775	2.3873	3.8197	4.7746	9.5493	14.3239	19.0986
13	0.6207	3.1035	4.9656	6.2070	12.4141	18.6211	24.8282
16	0.7639	3.8197	6.1115	7.6394	15.2789	22.9183	30.5577
19	0.9072	4.5359	7.2575	9.0718	18.1437	27.2155	36.2873
22	1.0504	5.2521	8.4034	10.5042	21.0085	31.5127	42.0169
25	1.1937	5.9683	9.5493	11.9366	23.8732	35.8099	47.7465
28	1.3369	6.6845	10.6952	13.3690	26.7380	40.1070	53.4761

5. Conclusion

The present static and dynamic analysis performed on the three-bladed H-Darrieus wind turbine based on 3D CAD FEA using industrial modelling and simulation software (SolidWorks), allowed finding significant mechanical and structural parameters. They permit to better understand the interaction between elastic, inertial and damping forces of the system during natural vibration modes. This dynamic design-optimisation process performed in development phase of the new product showed the importance of natural vibration modes predicted to obtain answers to the dynamic behaviours of the system in numerical and animation forms. The structural parameters allowed to assess the dynamic quality, and aided to construct the modal model of the system. From natural frequencies calculated and mode shapes predicted, the dynamic stability and reliability design-optimisation of the HDWT are shown since all performance indicators are acceptable by international structure dynamic standards. Moreover, the resonance do not happening in the structure if we use an adequate control filter. Finally, the 3D CAD, FEA procedure saved design-optimization resources, shortens product development cycle, verified the dynamic quality of the system before prototyping, and contributed to increase its control devices performances. Once, the (WT) behaviour against vibrations is known, we allowed selecting the best materials to construct the system, diminishing the upkeep costs and forecasting the (WT) behaviour, anticipating possible failure.

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