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Impact of Incremental Piecewise Linear Cost/Benefit Functions on DC-OPF Based Deregulated Electricity Markets

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Abstract- Optimal power flow is regarded as an essential look ahead tool for independent system operators and energy market operators to ensure reliable grid operation under normal and severe circumstances. For instance, the system operator will focus on power system performance, power quality while considering physical constraints and limits of different network components. On the other hand, a market operator will deal with bidding and market clearing mechanisms while ensuring optimal and efficient operation. Real-world data submitted by different market players are presented as piecewise linear functions composed of linear segments defined through quantity price pairs instead of the conventional polynomial quadratic functions. This paper proposes a piecewise linear cost/benefit model based on the incremental method that was presented as a Mixed Integer Programming (MIP) model and it was incorporated within a market-based DC-OPF problem. Results were analysed and discussed for a modified IEEE 14 bus test system operated under a deregulated market framework using the MIP model and the conventional polynomial quadratic functions. The General Algebraic Modelling System (GAMS) was used for problem formulation and simulation.

Keywords Deregulated electricity markets, General Algebraic Modelling System (GAMS), Mixed integer linear programming (MILP), Optimal Power Flow (OPF).

1. Introduction

Electricity markets around the world never ceased to evolve in order to accommodate novel technologies, that may lead to improved efficiencies and provide more choices while ensuring a reliable service. It worth noting that the introduction of Distributed Energy Resources (DER), active distribution networks and microgrids has pushed the market to alter its construction in order to take account for bidirectional energy paths, multilateral trading schemes and Peer-To-Peer (P2P) contract developed under a consumer concentric market design [1]. These technological developments have made the conventional market approaches less efficient and triggered the implementation of novel technologies that focus on the demand side. Meanwhile, electricity markets are evolving by introducing novel products or by giving birth to whole new entities [2,3], which in turn will create additional stress on the existing infrastructure and bring further computational complexity to the market clearing problem. It should be noted that power markets are unevenly deployed worldwide due to technical and regulatory complexity [4]. State of art models are based on a decentralized structure integrating the wholesale market with the retail market with the presence of power exchange contract and P2P contract [5].

The main idea behind the latest developments is to incite consumers to follow a competitive behaviour and stop treating power as a highly required commodity, which will prevent generating companies from ruling the electricity market [6]. In order to deal with renewable energy uncertainty and volatility combined to many micro grid operating challenges, the researchers tend to extract the demand side's flexible behaviour to support voltage profiles and manage network congestions. By the way, many European distribution operators have experienced voltage stability issues in zones with high renewable energy penetration [7]. Flexible demand and energy storage system are widely proposed in conjunction with DER in order to accommodate aforementioned energy unbalances and generation fluctuations [8,9,10]. Demand elasticity may be achieved through different demand side management strategies. Indeed, introducing load flexibility mechanisms may lead to increased efficiency and higher profits [6] associated to better economic operation of generating units especially wind turbines [11]. Moreover, many issues linked to security of supply may be accordingly resolved.

Mid-continent Independent System Operator (MISO), Pennsylvania-New Jersey-Maryland (PJM) interconnection, Nord Pool and Guangdong market in China and many other markets are requesting participants to send their offers and bids using Piecewise Linear Functions (PLF). New York Independent System Operator (NYISO) uses exclusively stepwise functions (Blocks), while accepting up to 11 price-power pairs. MISO and Nord Pool still having support for stepwise supply curves. New Zealand electricity market operator has been studying the adoption of piecewise linear curves in order to overcome paradoxal block rejection, market degeneracy and price volatility [12]. Market degeneracy phenomena arises when the market clearing mechanism issues a range of marginal prices instead of a unique marginal price for a given a quantity, as shown by Fig. 1.

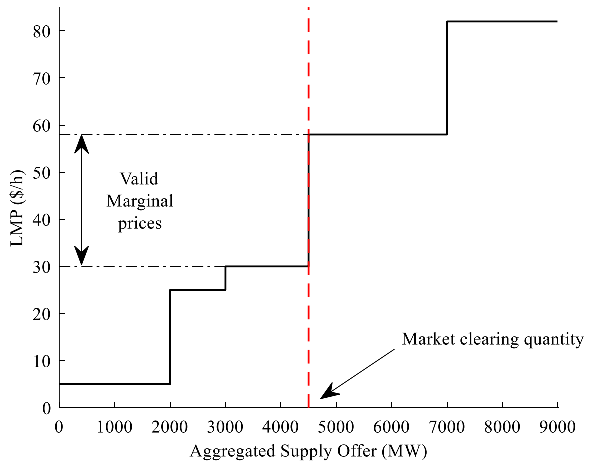

Fig. 1. Degeneracy phenomena perceived in energy markets that uses stepwise offer functions.

From a practical operational principle, Piecewise Linear Functions (PLFs) are accepted to be compatible with the physical characteristics of electricity generators. Indeed, production centres involving multiple units are well approximated using these curves, since there is a jump in the overall cost each time a unit is committed. Recent academic work dealing with hydropower future cost functions [13], hydropower production function [14-16], hydropower storage efficiency and energy decay and hydropower equivalent models [17] are making use of PLFs in order to facilitate interaction with balancing authorities and lowering computational complexity of unit commitment problems. A specific class of the unit commitment problem namely, network-constrained hydrothermal unit commitment (NCHTUC) incorporating hydropower production function (HPF) were approximated using the piecewise linear approach as discussed by [16]. This study considers the nonlinearities and forbidden zones of the HPF via aggregation of generating units while performing piecewise mixed-integer linear approximation.

Solar radiation forecast error cost has been effectively expressed as a convex PLF generated from various forecast methods [18]. Piecewise linear approximation (PWA) was exploited in [19] to model non-linear cost function for both solar thermal and heat storage technologies. The developed methods are applied to the optimal planning of a case study in Austria. Electricity storage plants arbitrage in the Day-Ahead Market (DAM) clearing problem considering real world price data derived from the Belgian power exchanges formatted as hourly piecewise linear curves was treated by [20]. A novel approach aimed to solve economic dispatch problem incorporating nonconvex cost functions has been elaborated in [21]. The overall problem has been transformed to several LP problems using an iterative piecewise linear function (PLF) approximation. Nonlinear AC optimal power flow problem has been formulated in conjunction with piecewise linear generation cost functions by [22]. An extensive numerical analysis across fifty-four realistic test cases illustrates that nonlinear optimization methods are surprisingly sensitive to the mathematical formulation of piecewise linear functions.

The need of efficient PLFs valuation methods is becoming significant, particularly for market-based OPF problems. In fact, incorporating PLFs within the optimisation problem will help aligning academic studies with real world applications and it will produce more realistic simulation results. Studies of load curtailment, load shedding, spinning reserve scheduling are still considering quadratic polynomial cost functions instead of PLFs which will be used practically by energy providers and balancing authorities. Despite the paramount importance of offer and bid forms in electricity markets, many prior papers seemed to focus exclusively on physical constraints linearization at the exception of a limited number of those that were dedicated to quadratic cost curve fitting techniques [23, 24], approximation error evaluation [25], error correction [26], and their impact on feasibility [27]. One should notice that [23, 24] are relying on the same theory of using SOS2 variables [28] to model convex piecewise approximation of polynomial nonlinear functions.

On the other hand, [25, 26] are treating the LP approximation of the cost function quadratic term.

It worth noting that none of the aforementioned work has treated the piecewise approximation of the social welfare objective function which involves simultaneously convex cost functions and concave benefit function. This objective function replicates a realistic open market, where the ISO tends to match supply offers with demand bids while ensuring reliable operation of the power system. In this work, a DC-OPF will be developed around an open economic framework while using both quadratic and piecewise linear form of the offer and bidding functions. Two locally ideal MIP models have been developed for each type of economic functions, weather its convex or concave. Curve construction, optimal curve fitting, bidding strategies are not treated in the present work.

2. Piecewise Linear Modelling Framework

Piecewise linear functions are being frequently introduced in different engineering optimisation problems to approximate nonlinear constraints and objective functions in order to achieve better computing performance in the expense of loss of accuracy. This type of functions admits multiple designations according to the field of application. For example, in statistics PLFs are commonly referred to as linear splines or linear spline regression, while in mathematics they are called polyhedral functions.

PLFs are composed of affine functions where each one of them describe a segment. For the rest of this paper, the term ‘breakpoint' abbreviated as *“Bp”* will be used to denote the point where these affine functions intersect. Generally, any univariate function defined on a delimited interval can be approximated through piecewise model. This interval is supposed partitioned into K segments with $K+1$ breakpoints, then the piecewise approximation of the function $f(x)$ can be described by Eq. (1):

$$f(x)=\begin{cases} m_1 x+d_1, & Bp_0 \le x \le Bp_1 \\ m_2 x+d_2, & Bp_1 \le x \le Bp_2 \\ \vdots & \\ m_K x+d_K, & Bp_{K-1} \le x \le Bp_K \end{cases} \tag{1}$$

where, m_1, m_2, ... m_k and d_1, d_2, ... d_k are the slopes and intercepts of the linear segments, respectively. The accuracy is affected by the number and the location of the breakpoint. Several methods have proposed to represent PLFs as a MIP model including multiple choice methods, convex combinations and incremental methods. Indeed, incremental method or the delta method has shown efficient behaviour especially in term of required continuous variables and its ability to produce a locally ideal model. The approach presented hereafter is founded on the study realised by [29].

2.1. Piecewise linear generation cost function

Supply functions C_G are convex increasing functions and they are approximated by the piecewise linear function $\tilde{C}_G$ defined by Eq. (2)

$$\tilde{C}_{G,t}\left(P_{G,t}\right)=\begin{cases} IC_{G,1}P_{G,t}+d^c_{G,1}, & P_G^{\min} \le P_{G,t} \le P_{G,1} \\ IC_{G,2}P_{G,t}+d^c_{G,2}, & P_{G,1} < P_{G,t} \le P_{G,2} \\ \vdots & \\ IC_{G,K}P_{G,t}+d^c_{G,K}, & P_{G,K-1} < P_{G,t} \le P_{G,K} \end{cases} \tag{2}$$

where, $P_{G,k}$ and $IC_{G,k}$ are provided by contractors as quantity (MW) - price ($/MWh) pairs for each generating unit. $d^c_{G,k}$ represents the segment intercept cost for generator G. $P_{G,t}$ is denoting the scheduled power output of the respective generating unit at time step t.

Allocated power supply is expressed as the sum of continuous variables $y^c_{G,k}$ as depicted by Eq. (3).

$$P_G = P_{G,K} + \sum_{k=1}^{K} y^c_{G,k} \qquad \forall k \in \{1,\dots K\} \tag{3}$$

Continuous variables must satisfy two sets of inequalities denoted by Eq. (4) and Eq. (5).

$$\begin{aligned} & y^c_{G,1} \le P_{G,K} - P_{G,K-1} \\ & y^c_{G,K} \ge 0 \end{aligned} \tag{4}$$

The second set of constraints is enforced using binary variables $\beta^c_{G,k}$ as follows

$$\begin{aligned} & y^c_{G,k} \ge (P_{G,K-k+1} - P_{G,K-k})\beta^c_{G,k}, \quad \forall k \in \{1,\dots K-1\} \\ & y^c_{G,k} \le (P_{G,K-k+1} - P_{G,K-k})\beta^c_{G,k-1}, \quad \forall k \in \{2,\dots K\} \end{aligned} \tag{5}$$

Discontinuous functions will give birth to the delta jump $\Delta^c_{G,k}$ defined at breakpoint by Eq. (6).

$$\Delta^c_{G,k} = \tilde{C}_G(P_{G,K-k}) - (IC_{G,K-k+1}P_{G,K-k} + d^c_{G,K-k+1}) \tag{6}$$

Finally, the supply function is expressed using continuous variables and jumps as follows.

$$\tilde{C}_G(P_G) = \tilde{C}_G(P_G^{\min}) + \sum_{k=1}^{K}(-IC_{G,K-k+1}y^c_{G,k} + \Delta^c_{G,k}\beta^c_{G,k}) \tag{7}$$

2.2. Piecewise linear demand benefit function

Demand functions B_{DL} are concave strictly increasing functions and they are approximated by the function $\tilde{B}_{DL}$ depicted by Eq. (8).

$$\tilde{B}_{DL,t}(P_{DL,t}) = \begin{cases} IB_{DL,1}P_{DL,t} + d^b_{DL,1}, & P^{\min}_{DL} \le P_{DL,t} < P_{DL,1} \\ IB_{DL,2}P_{DL,t} + d^b_{DL,2}, & P_{DL,1} \le P_{DL,t} < P_{DL,2} \\ \vdots & \\ IB_{DL,K}P_{DL,t} + d^b_{DL,K}, & P_{DL,K-1} \le P_{DL,t} \le P_{DL,K} \end{cases} \quad (8)$$

where, $P_{DL,k}$ and $IB_{DL,k}$ are provided by market players as quantity (MW) – price ($/MWh) pairs for each dispatchable (price responsive) load. $d^b_{DL,k}$ represents the segment intercept benefit for dispatchable load *DL*. $P_{DL,t}$ is denoting the dispatched demand at time step *t*.

Dispatched power demand is expressed as the sum of continuous variables $y^b_{DL,k}$ as follows

$$P_{DL} = P^{\min}_{DL} + \sum_{k=1}^{K} y^b_{DL,k} \qquad \forall k \in \{1,\dots K\} \quad (9)$$

Continuous variables must satisfy two sets of inequalities denoted by Eq. (10) and Eq. (11).

$$\begin{aligned} &y^b_{DL,1} \le P_{DL,1} - P^{\min}_{DL} \\ &y^b_{DL,K} \ge 0 \end{aligned} \quad (10)$$

The second set of constraints is enforced using binary variables $\beta^b_{DL,k}$ as shown by the equation hereafter.

$$\begin{aligned} &y^b_{DL,k} \ge (P_{DL,k} - P_{DL,k-1})\,\beta^b_{DL,k}, \quad \forall k \in \{1,\dots K-1\} \\ &y^b_{DL,k} \le (P_{DL,k} - P_{DL,k-1})\,\beta^b_{DL,k-1}, \quad \forall k \in \{2,\dots K\} \end{aligned} \quad (11)$$

Discontinuous functions give birth to the delta jump $\Delta^b_{DL,k}$ parameter expressed by Eq. (12).

$$\Delta^b_{DL,k} = \tilde{B}_{DL}(P_{DL,k}) - (IB_{DL,k}P_{DL,k} + d^b_{DL,k}) \quad (12)$$

Finally, the benefit function is expressed using continuous variables and incremental jumps as follows

$$\tilde{B}_{DL}(P_{DL}) = \tilde{B}_{DL}(P^{\min}_{DL}) + \sum_{k=1}^{K}(IB_{DL,k}y^b_{DL,k} + \Delta^b_{DL,k}\beta^b_{DL,k}) \quad (13)$$

3. Optimisation Problem Formulation

The conventional Direct Current - Optimal Power Flow (DC-OPF) is used to interpret the behaviour of the transmission network. The DC-OPF is a linearized version of the full OPF except the objective function that constitutes the only part incorporating inherently quadratic components rising from the generation cost functions, thus making the overall problem quadratic. In other words, dealing with objective function non linearities may lead to computational improvements and may enable the use of more mature and robust linear programming solvers.

In order to study the impact of PLFs on the model complexity, solution quality and computational performance, a deregulated economic environment has been formulated using both the quadratic and the piecewise linear from of the objective function. Simulation scenario is taking account of flexible demand behaviour and it will reflect the interaction with market prices while maximising the system operator social welfare.

Objective functions, equality and inequality constraints that constitutes the optimisation problem are discussed hereafter.

- Maximum social welfare objective function

In deregulated power markets, the independent system operator is responsible for grid operation and security whilst the market operator will guarantee efficient grid operation by maximising consumer benefit and minimising generation cost which is translated by the expression denoted by Eq. (14).

$$\begin{aligned} SW_t &= \max\left(\sum_{DL=1}^{N_{DL}} B_{DL,t} - \sum_{G=1}^{N_G} C_{G,t}\right) \\ &= \max\left[\begin{aligned}&\sum_{DL=1}^{N_{DL}}\left(a^b_{DL}P^2_{DL,t} + b^b_{DL}P_{DL,t} + c^b_{DL}\right) - \\ &\sum_{G=1}^{N_G}\left(a^c_G P^2_{G,t} + b^c_G P_{G,t} + c^c_G\right)\end{aligned}\right] \end{aligned} \quad (14)$$

where, a^b_G, b^b_G, c^b_G are quadratic polynomial coefficients of the consumer benefit function and a^c_G, b^c_G, c^c_G represent quadratic polynomial coefficients of generation cost function.

The main objective function *F* is expressed by the Eq. (15).

$$F = \max\sum_{t=1}^{st}\left(SW_t - \sum_{W=1}^{N_W} C_{W,t} - \sum_{W=1}^{N_W} P^C_{W,t}.VCW\right) \quad (15)$$

where, $C_{W,t}$ represents the wind generation cost and $P^C_{W,t}$ represents the wind power curtailment and *VCW* is the value of curtailed wind energy in $/MWh.

- Wind generation cost function

$$C_{W,t} = d_{W,t}P_{W,t} \quad (16)$$

where, $d_{W,t}$ is the negotiated day ahead price of wind generation and $P_{W,t}$ is the wind energy injection at bus *W* and at time step *t*.

- Reference bus voltage angle

$$\delta_{ref} = 0 \quad (17)$$

This constraint is setting the phase reference for the system.

- Nodal power balance equation

$$P_{G,t} + P_{W,t} - P_{L,t} - G_{sh} - (Cf - Ct)^T B_{ff} (Cf - Ct)\delta_{i,t} + (Cf - Ct)^T \theta_{Br} b_{Br} = 0 \tag{18}$$

where, G_{sh} is the conductance shunt matrix, B_{ff} is a portion of the admittance branch matrix expressed by Eq. (26) and Eq. (29), θ_{Br} is the phase shifting angle at branch number *Br* and b_{Br} is the branch susceptance.

$P_{G,t}, P_{W,t}, P_{L,t}$ are the active power generated by conventional generators, the active power injected by wind generators, and the active power consumed at respective bus ID and at time step *t*.

- Branch power flow equation

$$P_{ij,t} = b_{Br}(\delta_{i,t} - \delta_{j,t} - \theta_{Br}) \tag{19}$$

where, $P_{ij,t}$ is the active power flowing from bus *i* towards bus *j*, $\delta_{i,t}$ and $\delta_{i,t}$ are the voltage angles of the respective bus at time step *t*. b_{Br} is the branch susceptance expressed by Eq. (30)

- Generating unit operational limits

$$P_G^{\min} \le P_{G,t} \le P_G^{\max} \tag{20}$$

where, $P_G^{\min}$ and $P_G^{\max}$ are defining the lower and the upper generator limit. $P_{G,t}$ is the active generation output scheduled at time step *t*.

- Generating unit ramping rates

$$\begin{aligned} P_{G,t-1} - P_{G,t} &\le R_G^{Down} \\ P_{G,t} - P_{G,t-1} &\le R_G^{Up} \end{aligned} \tag{21}$$

where, R_G^{Down} and R_G^{Up} are the lower and the upper ramping rates for conventional generator *G*.

- Wind energy availability

$$0 \le P_{W,t} \le A_{W,t} \tag{22}$$

This constraint is defining the variation range of wind injection $P_{W,t}$. Where, $A_{W,t}$ is the available wind power at time step *t*.

- Wind energy balance

$$P_{W,t} + P_{W,t}^C \le A_{W,t} \tag{23}$$

where, $P_{W,t}^C$ is the amount of curtailed or undispatched wind energy at time step *t*.

- Branch power flow limits (Thermal limit)

$$P_{ij,t} \le P_{ij}^{\max} \tag{24}$$

- Demand flexibility constraint

In order to consider the price responsive demand, the flexibility range is set to each dispatchable load using the constraints below

$$P_{DL}^{\min} \le P_{DL,t} \le P_{DL}^{\max} \tag{25}$$

In case of load curtailment engagement, the load power factor is supposed to be kept constant. It worth noting that interruptible demand is one of the most common load management programs used by the ISO to deal efficiently with power unbalances and severe congestions. The customer signs a contract with the ISO to reduce the demand as and when requested.

- Branch susceptance matrix

$$B_{Br} = \begin{bmatrix} B_{ff} & B_{ft} \\ B_{tf} & B_{tt} \end{bmatrix} = b_{Br} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{26}$$

$$B_f = \left[B_{ff} \right] Cf + \left[B_{ft} \right] Ct \tag{27}$$

$$B_t = \left[B_{tf} \right] Cf + \left[B_{tt} \right] Ct \tag{28}$$

$$B_{Br} = Cf^T \left[B_f \right] + Ct^T \left[B_t \right] \tag{29}$$

Connection matrices *Cf* and *Ct* used in building the branch admittance matrices are equal to 1 for each branch *Br* connecting bus *i* to bus *j*. otherwise, the rest of their elements is equal to zero.

b_{Br} is defined in terms of the series reactance x_{Br} and tap ratio τ_{Br} for branch.

$$b_{Br} = \frac{1}{x_{Br}\tau_{Br}} \tag{30}$$

In order to implement the MIP approach in the main optimisation algorithm, one should simply replace the quadratic cost and benefit of Eq. (14) by the appropriate PLF model. Integrating this MIP model will result in a MILP problem.

4. Piecewise Linear Modelling Framework

In order to study the impact of using piecewise linear function on the OPF solution several simulations were conducted on a modified IEEE 14 test. Numerical simulations were executed in the General Algebraic modelling System (GAMS) environment using an appropriate solver. The test hardware was running Windows 10 64 bits version operating system on an AMD A8-7410 @ 2.5 GHz with 8GB of DDR3 RAM.

GAMS/IBM ILOG CPLEX solver was chosen to provide solution to the optimisation problem whether it's a Quadratic Problem or Mixed Integer Linear Problem. The same solver configuration was used for the entire test process. The dual simplex method was specified through *lpmethod* and *qpmethod* GAMS/CPLEX options. Modified solver options are summarized in table 1in Table 1.

Table 1. Modified solver options

Solver option	Value
aggind	1
Prepass	1
Probe	-1

The modified IEEE 14 is composed of 5 Generating Companies (GenCos) and 11 Distribution Companies (DisCos), 20 transmission lines, 3 on load tape changing transformers and one shunt at bus 9. Case peak active power demand is increased by 50% while preserving original power factor. Voltage limits were adjusted to 1.05 and 0.95 for the upper and the lower bound respectively. A wind farm with a rated capacity of 100 MW is connected to the 9th bus. The wind farm is assumed to operate at unity power factor. The number of segments K is supposed equal to 7.

The capability of the proposed approach is illustrated by comparing optimal power flow results obtained with polynomial quadratic functions against those obtained with piecewise linear models. Results are presented for both QP and MILP formulation of the DC-OPF discussed above for a power base of 100 MVA and bus 1 selected as reference bus. Fixed loads located at bus 4, 5, 9, 10, 11, 12, 13, 14 were converted to price responsive demands with a predefined flexibility of 20%. The same power factor was preserved for the entire operating interval (i.e., active and reactive power are cut by the same proportion). Due to licensing limitation only 18 hours of simulation has been achieved for the MILP formulation.

4.1. Model Convergence

This subsection is dedicated to discuss convergence characteristics and computational performance analysis. Fig. 2, demonstrates the convergence characteristic of the elaborated optimisation problem. Main model statistics have been regrouped in Table 2.

It is clear that the MILP formulation have generated more equations and variables compared to the QP formulation. It should be noted that for a cost or benefit function that is indexed to one variable and partitioned into K segments, the incremental method requires K continuous

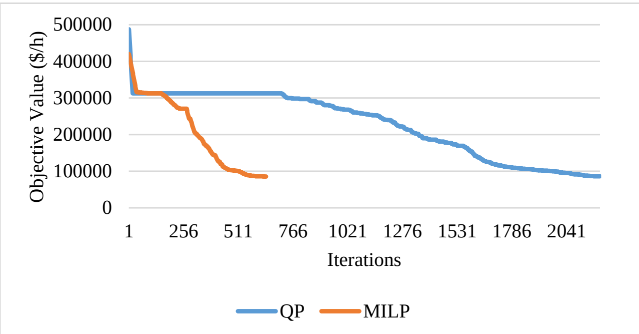

Fig. 2. Convergence characteristics of QP and MILP formulation of the proposed model

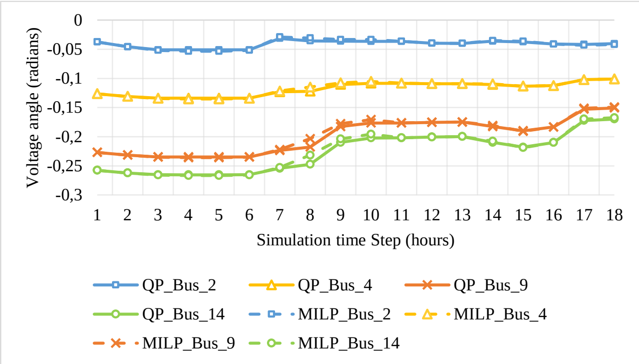

Fig. 3. Bus voltage angle profiles

variables and K -1 binary variables. The MILP model took longer to solve the problem, even it requires less iterations to attain optimality as illustrated by Fig. 2. This time delay is mainly due to pre-processing operations executed by CPLEX before starting the solution algorithm which will be compensated for larger test cases.

Table 2. Model Statistics

Problem Formulation	QP	MILP
Number of variables	1207	4483
Number of equations	1081	4915
Non-zero elements	3555	13635
Iteration count	2196	640
Completion time (s)	0.084	0.271
Number of variables to time ratio ($\times 10^{-5}$)	6.9594	6.0450

4.2. Load Flow Results

In this subsection, load flow results are provided as follows; Voltage angles for selected buses are depicted in the Fig. 3. Generators' output profiles are illustrated in Fig. 4. Injected and curtailed wind power are presented by Fig. 5 and Fig. 6, respectively. Branch flows are depicted in Fig. 7.

From the figure above, it can be stated that voltage angle profiles produced by the MILP formulation are very similar to those produced by QP formulation.

As it can be seen from the Fig. 4, generators 3, 6 and 8 have generated the same amount of power. Generators 1 and 2 are approximated in a complementary manner (an increased generation output of generator 1 results in a decrease in generator 2 output and vice versa). This

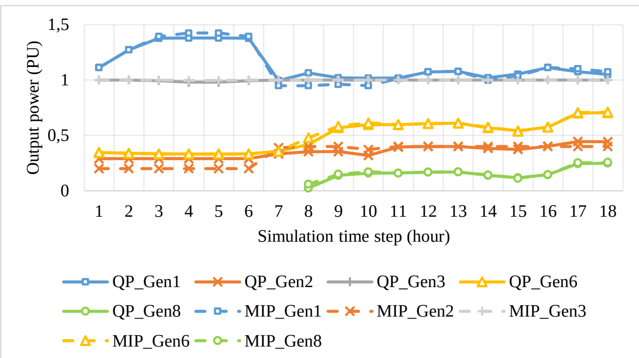

Fig. 4. Generators' output profiles

mismatch in output power may be attributed to the fact that the MILP model tend to fix generators' operating point on their break points.

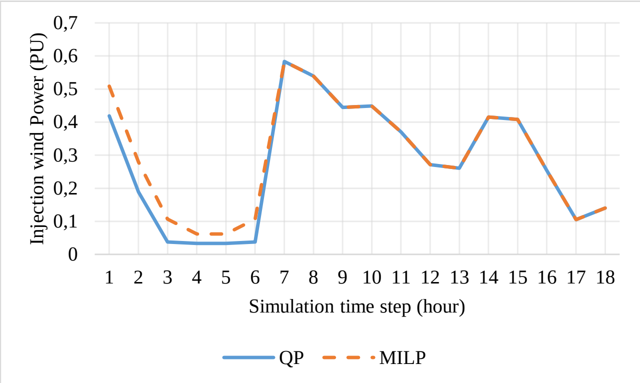

Fig. 5. Injected wind power profile at bus 9.

As shown in Fig. 5, MILP formulation resulted in a maximised wind energy injection throughout the simulation process on the other hand wind power curtailment is kept at its minimum as shown in the Fig. 6. It should be noted that wind curtailment has been engaged for the first six hours while demand was relatively low and it was supplied mainly with conventional generators.

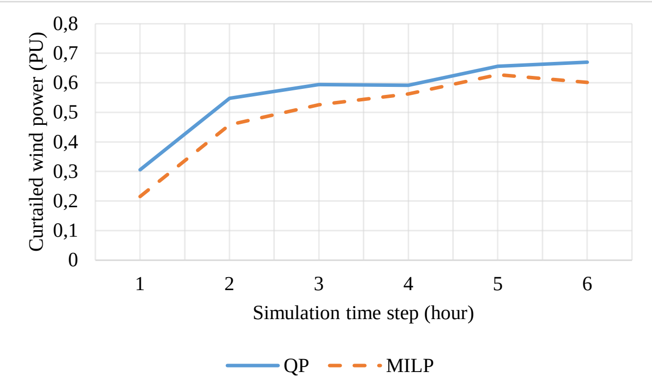

Fig. 6. Curtailed wind generation for the first six hours.

The formulation based on the piecewise linear model has produced identical power transactions to those generated based on the quadratic model, as demonstrated by Fig. 7.

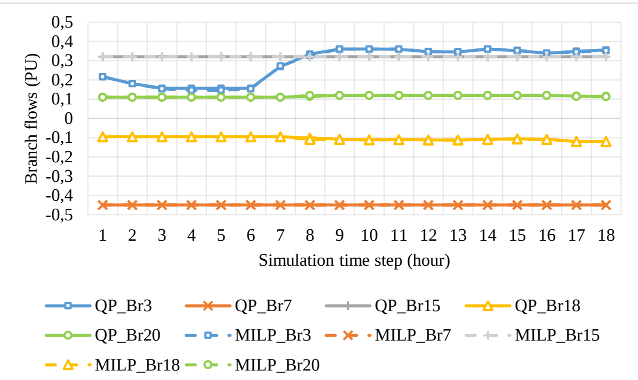

Fig. 7. Transmission branch flow profiles

4.3. *Market Prices*

In this subsection, results based on MILP model are compared to those produced by the QP model from an economical point of view. Fig. 8, illustrates the variation of locational marginal prices throughout the simulation process. Table 3 and 4 regrouped the standard deviation values based on QP and MILP formulation. Fig. 9, represents objective function values.

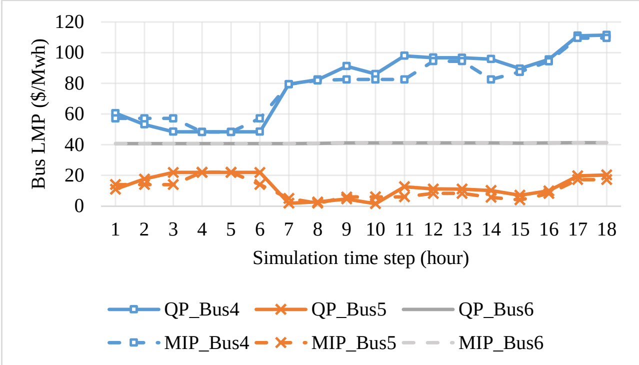

Fig. 8. Bus locational marginal prices variation

Different locational marginal price patterns have been produced by MILP formulation for the 4th and the 5th bus while we had a perfect match for bus 6.

The standard deviation parameter is employed to determine the price deviation from LMP mean value. Higher values of the standard deviation parameter may lead to volatile market. MILP based formulation produced inferior price dispersion on selected buses and time steps, as demonstrated in Table 3 and 4. This may be translated to a more consistent and stable market.

Table 3. Standard deviation at specific bus

Formulation	QP	MILP
Bus 4	22.56	19.51
Bus 6	0.28	0.27

Table 4. Standard deviation at specific time step

Formulation	QP	MILP
4^{th} step	12.68	12.64
14^{th} step	35.43	33.04

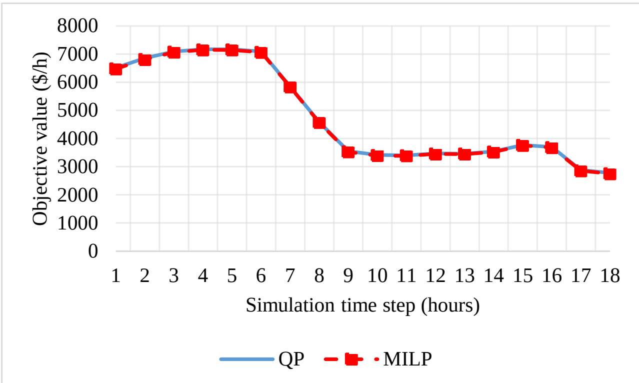

Fig. 9. Objective function hourly values using QP and MILP formulation

As shown in Fig. 9, MILP formulation has generated a good match of objective values for different loading conditions.

5. Conclusion

In this paper, Piecewise linear functions have been introduced to the DC-OPF as a MIP model using the incremental method. The overall optimisation algorithm was formulated as a MILP problem and it was solved by GAMS/CPLEX. Results generated by quadratic cost/benefit functions were compared to those generated by piecewise linear cost/benefit functions for a deregulated market framework. It has been found that the MILP version of the DC-OPF has succeeded to produce acceptable solutions while ensuring maximum social welfare, although it requires more variables.

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