# Optimal sizing of a Stand-alone photovoltaic system using statistical approach

Safa Fezai*,**‡ , Jamel Belhadj *,**

*University of Tunis, ENSIT, EED, BP 56, Montfleury 1008, Tunis, Tunisia.

**University of Tunis ElManar, (LSE) ENIT BP 37, Le Belvédère 1002 Tunis, Tunisia.

(fezai_safa@live.fr , Jamel.Belhadj@esstt.rnu.tn )

‡ Corresponding Author; Safa FEZAI, University of Tunis ElManar, (LSE) ENIT BP 37, Le Belvédère 1002 Tunis, Tunisia, fezai_safa@live.fr

*Received: 03.03.2014 Accepted: 11.04.2014*

**Abstract-**The intermittent nature of renewable resources remain one of the major impediments to the evolution of electricity production from renewable energies. This paper presents a methodology to optimally size a photovoltaic (PV) stand-alone system with battery storage. Its originality resides in taking into account the stochastic nature of the solar radiation in the sizing process by using statistical techniques. The optimal design is based on the inverse conception, which gives the optimum number of photovoltaic panels and the optimum number of batteries. The inputs of the optimization process are only the radiation data and the load demand. The radiation data of the location were synthesized by the statistical technique from which the average hourly PV power of the day of each month was determined. A photovoltaic panel that most fits the site was chosen among a list of four panels with different characteristics to be afterward deployed for sizing. The loss of power supply probability (LPSP), related to the system reliability was used to evaluate the efficiency of the sizing methodology. The number of PV panel and batteries obtained from the system simulation during 24h presented an acceptable rate of load satisfaction.

**Keywords** Stand-alone PV system, statistical approach, distribution function, optimal sizing, LPSP.

## 1. Introduction

The current states of the environment and fossil fuels confirmed both the previous predictions of the necessity to find new sources of energy. In fact, the deficit in fossil fuels is due to the excessive use of the conventional power, more than 80 per cent of our global energy comes from fossil fuels, which causes the global warming [1].

The observable greed to electrical energy is taking the biggest share of this situation; meanwhile, more than 1.5 billion people still lack access to electricity. This uneven balance makes renewable energy the adequate solution to the energy needs of current and future generations even in the most remote areas all over the world [1,2].

The use of renewable energy for the production of electricity has evaluated for the last years, especially PV production achieving almost 30 per cent of new generation in 2011. This evolution came along with new problematic such as finding the optimal sizing of each component of the system. It is necessary to determine the adequate combination (number of PV panels and batteries) for sufficient energy production to cover the load demand. The stochastic nature of the energy resource, whether it is the irradiance or the wind speed or both for hybrid PV/Wind systems, influences the design of the system [3-7].

Several optimization techniques for PV, wind or wind/PV systems with energy storage have been studied. Borowy and Salameh [6,7] have developed some methodologies for optimally sizing a wind/PV system with and without a battery bank for a given load. Different algorithms were used: in [8] a methodology for the calculation of the sizing and optimization using a deterministic algorithm. The objective of this methodology is the minimization of the total costs and guaranteeing the availability of the energy which are the same objectives in [9] using genetic algorithm and the LPSP( Loss of Power Supply Probability) term referring to the satisfaction of the load demand.

In this paper, a stand-alone PV system with batteries for the supply of a farm in a remote area in the center west of Tunisia was studied. Starting only with the solar radiation data and the load profile, the number of PV panels and the number of the storage batteries was optimally determined. The design of the studied system is shown in fig.1.

The proposed sizing methodology consists on the determination of the average power output, which depends on the distribution function of the solar data and the PV module power output. Statistical techniques were used to determine the solar data distribution function and the PV panel that most fits the location.

For the system simulation, load demand and statistical solar profile were use. The validation of the simulation's results was based on the loss of power supply probability (LPSP) rate. The results of this evaluation are presented and discussed.

## 2. Sizing methodology

### 2.1 Solar profile model

The use of the statistical technique for modeling the solar profile permit computing the average power generated in the site. Its expression is given by the following equation [7].

$$P_{avg} = \int P(S) \cdot f(S) \cdot dS \qquad (1)$$

Where:
P(S): the PV module power output,
$f$(S):irradiance probability density function.

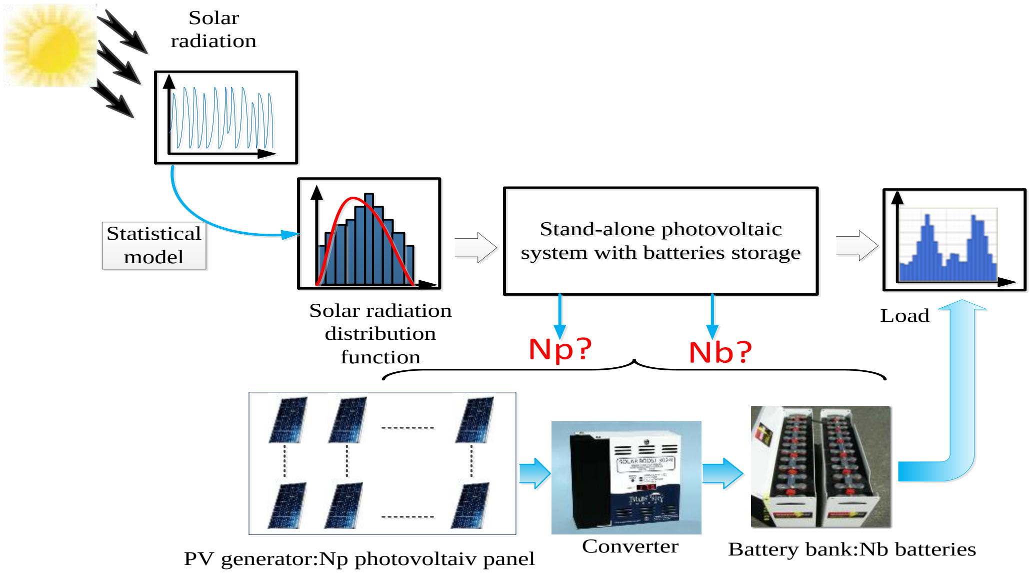


**Fig. 1.** Stand-alone photovoltaic system design

Therefore, for the calculation of the average power output we need to determinate the irradiance probability function and the PV module power output for every level of solar irradiance in every hour of a typical day for every month throughout the year.

#### 2.1.1 Solar irradiance distribution function

The irradiance data have usually a bimodal distribution function. The data of the typical day is divided into two groups each one have a unimodal distribution function. To describe the random phenomenon of the irradiance data three distribution functions were used i.e., Beta, Weibull and Log-Normal. Every group was matched to each distribution function named previously as shown in the example in fig.3. Then we use the kolmogorov-Smirnov test to determinate the distribution function that most fits the distribution of the irradiance data [6, 7].

#### 2.1.2 The PV module power output

To determine the PV module power output, we use the equivalent circuit of a PV cell. It is presented in fig.2. In fact, the module power output is equal to the product of the module output voltage and the output current [10,11].

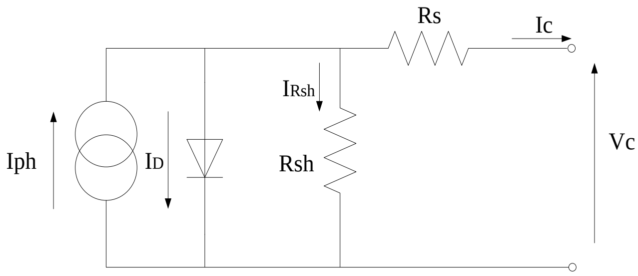


**Fig. 2.** Equivalent circuit of a solar cell

The PV panel is an association of $N_p$ cells in parallel and $N_s$ cells in series, so the module output voltage and output current are expressed as follows [12]:

$$V_P = N_S \times V_C \quad (2)$$

$$I_P = N_P \times I_C \quad (3)$$

Where $V_c$ : Cell output voltage
$I_c$ : Cell output current

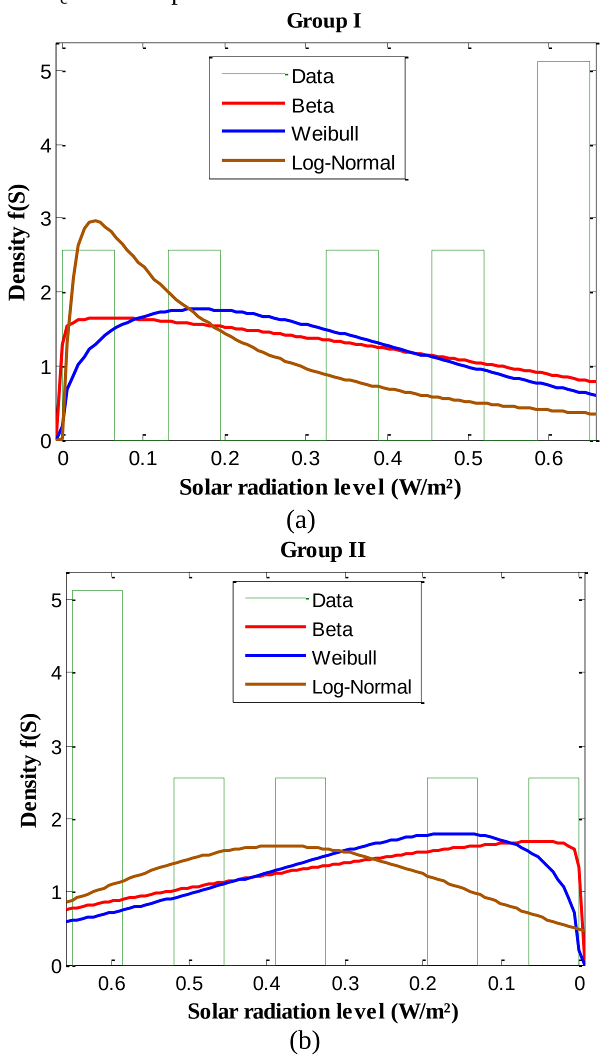


**Fig. 3.** Histogram and probability density functions versus solar radiation level for a typical day of December for group I (a) and group II (b)

*2.2 PV module choice*

The selection of the adequate PV panel was based on the Capacity Factor "CF" which evaluates the performance of the PV module. It is defined as the ratio between average power output given by (equation 1) and rated power for the considered module $P_{mp}$ [13].

$$CF = \frac{1}{P_{mp}} \int P(S) \cdot f(S) \cdot dS \quad (4)$$

To choose the optimal PV module for the studied site, we computed the capacity factor for four different PV modules presented in table 2, following these steps:

a. Choose a month from each season,
b. Determinate the distribution function for every typical day of the month in every season,
c. Calculate the Capacity Factor for each PV panel from the list,
d. Calculate the annual Capacity factor.

The results of steps "a" and "b" are given in table 1.

**Table 1.** The probability distribution function for every month

| | December | March | July | September |
|---|---|---|---|---|
| **Group I** | Beta | Beta | Weibull | Weibull |
| **Group II** | Beta | Beta | Weibull | Weibull |

The PV panel with the higher capacity factor is the one that provides the higher energy production in the location for the studied stand-alone PV system. Having the upper CF, the module A is the adequate one for the site as shown in table 3

## 3. Sizing algorithm

Once the PV module is selected, a proper much between the installed capacities with the isolated load is essential to optimize the performances of such application.

*3.1 Load demand profile*

The site under consideration is situated in the center-west of Tunisia (latitude: 35° North, longitude: 9.5° East) and it is characterized by an important solar radiation potential [14]. We took the case of a typical load demand of a farm.

The profile of a typical load demand for a day is given in fig.4. The highest power levels claimed by the load are at 6 and 8 am and at 5:30 and 9 pm. The consumption peaks are related to the inhabitant's attitudes. The first peak takes place because inhabitants wake up and get ready to go out (work, school). Then the energy demand begins to stabilize. The second peak is due to the presence of inhabitants in the house. After 9 pm, returns the stabilization progressively because residents go to sleep.

**Table 2.** PV panels list

| | Panel A | Panel B | Panel C | Panel D |
|---|---|---|---|---|
| **Pmp (W)** | 95 | 180 | 160 | 240 |
| **Vmp (V)** | 18.43 | 35.2 | 36 | 30.2 |
| **Imp (A)** | 5.16 | 5.11 | 4.44 | 7.95 |
| **Voc (V)** | 21.98 | 43.6 | 44 | 37.3 |
| **Isc (A)** | 5.64 | 5.5 | 4.7 | 8.63 |
| **α (A/°C)** | +0.03%/°C | +0.10%/°C | +(0.065±0.015)%/°C | 0.030%/°C |
| **β (V/°C)** | -0.46%/°C | -0.38%/°C | -(0.16±0.01)%/°C | -0.330%/°C |

**Table 3.** Annual Capacity factor

| | December | March | July | September | Annual CF |
|---|---|---|---|---|---|
| **Panel A** | 0,162 | 0,279 | 0,283 | 0,314 | 0,260 |
| **Panel B** | 0,162 | 0,273 | 0,278 | 0,308 | 0,255 |
| **Panel C** | 0,157 | 0,266 | 0,271 | 0,300 | 0,248 |
| **Panel D** | 0,160 | 0,268 | 0,273 | 0,302 | 0,251 |

*3.2 Storage battery characteristic*

The battery is an important element in stand-alone PV systems. Actually, the energy stored in the battery can be the supplying energy during daytime with none or low solar energy production, and during nighttime. The performance of batteries is complicated and hard to predict because of the uncontrollable charge and discharge cycles in such systems. Usually, manufacturers specify round trip efficiency and a depth of discharge [4,7]. The behavior of battery storage can be modeled according to the available produced PV energy. Two different cases arise: the first case when the load is satisfied and the extra energy is devoted to the battery charging, the second case consists in using the energy stored in the batteries to assure covering the deficit [7]. The process of charging and discharging can be modeled by the following equations respectively equation (5) and equation (6):

$$E_B(t) = E_B(t-1) + \left(E_{PV}(t) - E_{Load}(t)\right)\eta_{bat} \qquad (5)$$

$$E_B(t) = E_B(t-1) - \left(E_{Load}(t) - E_{PV}(t)\right) \qquad (6)$$

Where:

$E_B(t)$ : energy stored in the batteries at the hour “t”,

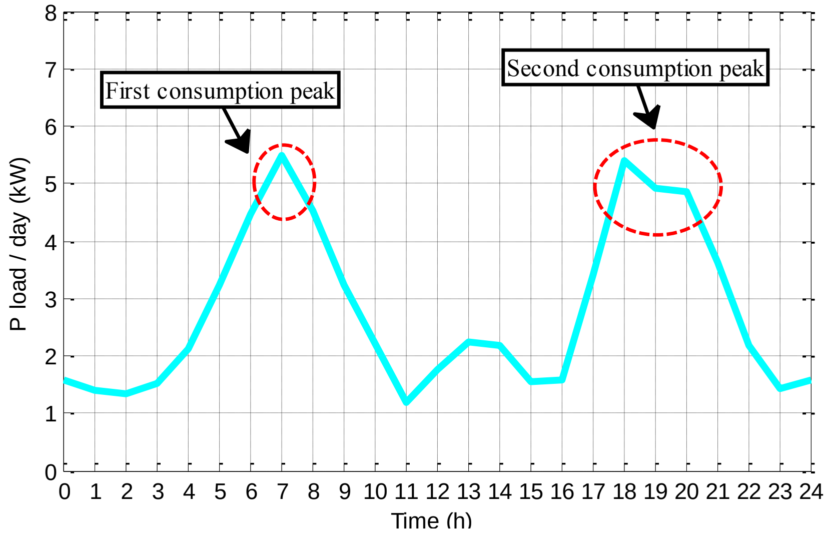


**Fig. 4.** Farm load profile for one day

$E_B(t-1)$: energy stored in the batteries in a previous hour "t-1".

$E_{PV}(t)$: Energy produced by the PV generator at the hour "t".

$E_{Load}(t)$: Load demand Energy.

$\eta_{bat}$: round trip efficiency of the batteries.

The table bellow gives the characteristics of the battery used in this study:

**Table 4.** Characteristics of the battery

| | |
|---|---|
| **Nominal Voltage** | 12 (V) |
| **Nominal capacity** | 100(Ah):Ampere hour |
| **Round-trip efficiency** | 85% |
| **Depth Of Discharge DOD** | 80% |

*3.3 Simulation program*

The aim of the proposed methodology is the computation of the number of PV panels and batteries in the battery bank having as initial data the radiation statistical profile and the load demand profile. It is primordial to take into account the constraints of the system's components. In this context, an interest is dedicated to the battery's operating constraint. Energy in the batteries has upper and lower limits that must be respected. The state of charge (SOC) is a referent symbol to the amount of energy stored in batteries and it was taken into account as explained by equation (7) and equation (8).

The energy stored in the battery must be bounded by two limits: a maximum energy SOCmax not allowed to exceed and a minimum level of energy SOCmin not to go under. To insure the respect of this constraint, the energy stored in the battery was compared to the allowed energy levels, which protects the battery from being damaged [7].

$$E_{B\min} \le E_B(t) \le E_{B\max} \tag{7}$$

$$E_{B\min} = \left(1 - DOD\right) \times C_{bat} = SOC \times C_{bat} \tag{8}$$

Where:

DOD: depth of discharge of the battery,

$C_{bat}$: nominal capacity of the battery,

SOC: state of charge of the battery.

The flowchart (figure 5) represents the program used for the computation of the optimal number of PV panels and batteries. The input data for this program are: a fixed minimal and maximal number of PV panels and batteries and the nominal capacity of the battery.

The energy generated by the PV models $E_G(t)$ given by equation (9) is compared at every hour with the energy demand of the load.

$$E_G(t) = N_{PV} \cdot E_{PV}(t) \tag{9}$$

Where:
$N_{PV}$: the number of PV models,
$E_{PV}(t)$: the energy generated by a PV panel depending of the $P_{avg}$ given by equation (1).

**4. Results and discussions**

The average power generated by the PV generator was based on statistical techniques; its influences appeared in the solar radiation modeling and the choice of the adequate PV panel.

The simulation of the system's functioning permits the determination of the appropriate PV panels' number and batteries number for the considered location. Table 5 represents the results of the sizing algorithm.

**Table 5.** Optimal number of PV panels and batteries

| | | |
|---|---|---|
| Number of PV panels | $N_{PV}$ | **24** |
| Number of batteries | $N_{BAT}$ | **28** |

PV generator consists of a number of PV panels connected in series $N_{PV,S}$ and in parallel $N_{PV,P}$. The battery bank is also constituted by a number of batteries in series $N_{BAT,S}$ and in parallel $N_{BAT,P}$.

The PV generator and the battery bank both debits on a fixed DC bus Voltage. It is fixed at 48 Volts in this case of study. It is the standard value respecting safety conditions. So, the number of PV panel and batteries associated in series can be determined respectively by equation (10) and equation (11) [10,15]:

$$N_{PV,S} = \frac{V_{BUS}}{V_P} \tag{10}$$

$$N_{BAT,S} = \frac{V_{BUS}}{V_B} \tag{11}$$

Where:

$V_{BUS}$: the DC bus voltage,

$V_P$: PV panel's nominal voltage,

$V_B$: Battery's nominal voltage.

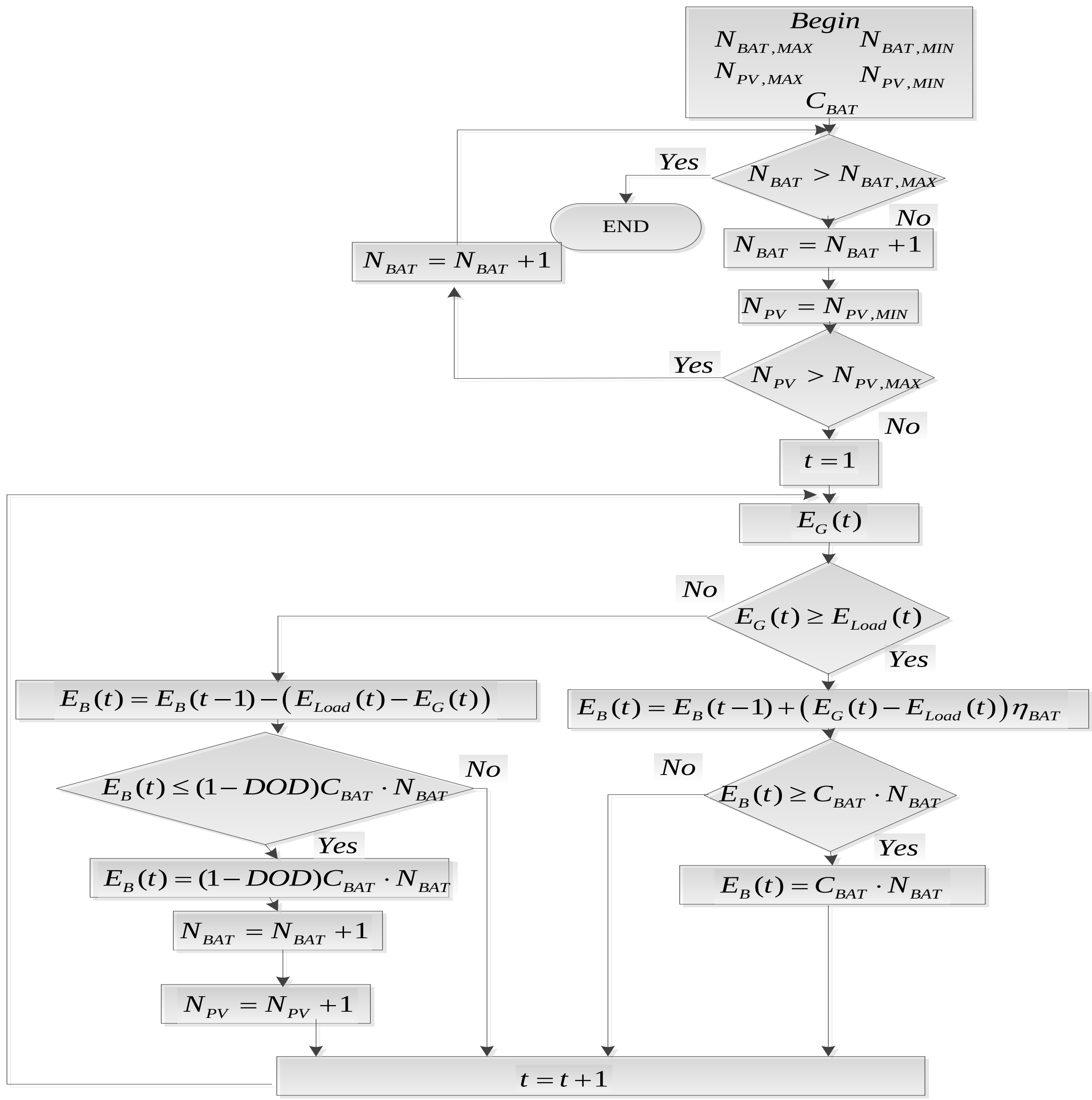


**Fig. 5.** Sizing procedure program

Unlike the elements connected in series, which can be determined just by knowing the DC bus voltage. The number of PV panels and batteries connected in parallel are the subject of optimization. In fact, minimization of the global number of PV panels and batteries leads to the minimization of the elements connected in parallel. Equation (12) and equation (13) allow the determination of the number of PV panels and batteries correspondingly [10]:

$$N_{PV} = N_{PV,S} \times N_{PV,P} \tag{12}$$

$$N_{bat} = N_{bat,s} \times N_{bat,P} \tag{13}$$

For the evaluation of the obtained results, Loss of Power Supply Probability LPSP is defined by equation (14). This term is used for the determination of the reliability of the system in regards of the computed number of PV panels and batteries. In effect, LPSP gives an idea on whether the load is satisfied or not: if LPSP is equal to 1 so the load will never be satisfied and if LPSP is equal to 0 it means that the load will always be satisfied [9,10].

$$LPSP = \Pr\left\{E_B(t) \leq E_{B\min} ; t \leq T\right\} \tag{14}$$

The algorithm used for identifying the efficiency of the proposed methodology is explained by the flowchart in fig.6. The inputs of the evaluation algorithm using LPSP are the computed number of PV panels and batteries and the nominal capacity of the battery. Batteries are considered initially charged.

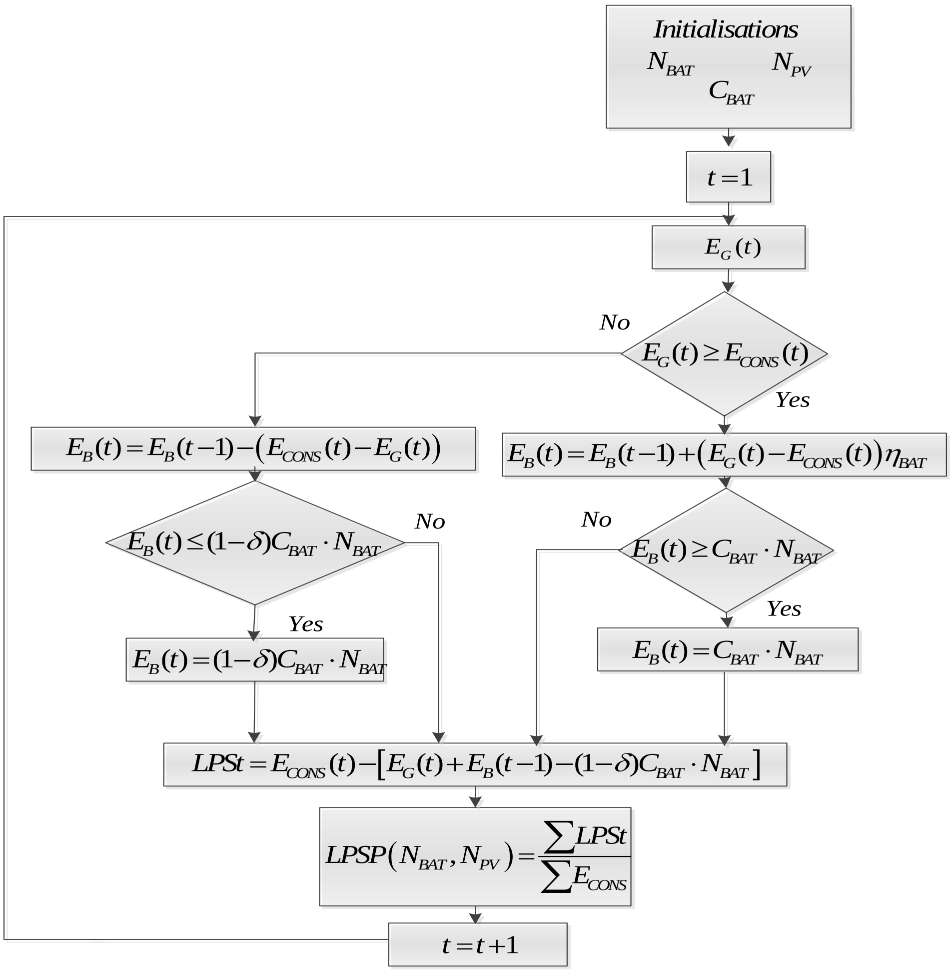


**Fig. 6.** LPSP calculation diagram

The results of this simulation are shown in fig.7. The load is satisfied along the entire day. The loss of power supply probability is equal to 0; which means that the load is satisfied. Furthermore, this algorithm insures the respect of the constraints related to the energy stored in batteries. During daytime, PV generator is producing electricity. That is why the batteries attain the maximal allowed energy Ebmax. The minimal allowed energy is not attaining in this time duration.

The satisfaction of the load confirms the efficiency of the methodology proposed in this paper. Taking into account the stochastic feature of the solar radiation by using the statistical techniques conducted to an optimal sizing of the stand- alone PV system for a specified location.

For the same computed numbers of elements (Nbat=28 and NPV=24), the efficiency of the system was evaluated for a two days duration. The results are given in fig.8.

The maximal and minimal constraints are respected in this case. The circled areas are referring to hours when the batteries are discharged and the PV generator cannot produce electricity (during the night).

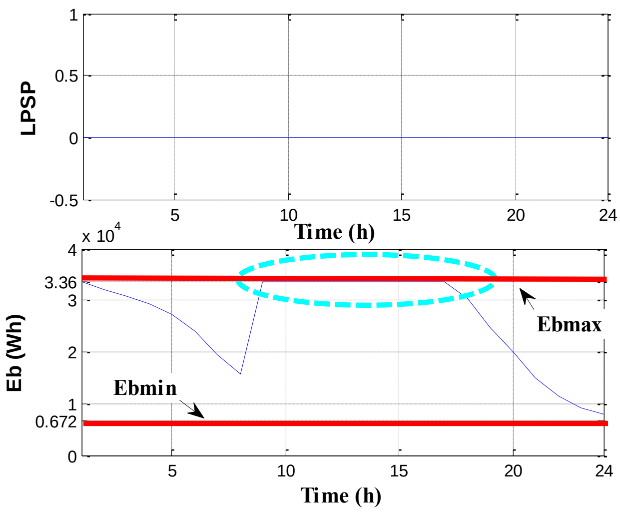


**Fig. 7.** LPSP and Energy stored in batteries Eb (Watt hour) of battery bank during a day

Although the number of elements is corresponding to a one-day simulation, the number of hours in which the LPSP is equal to 1 is tolerable. In fact, for a period of 48 hours, the value of LPSP is equal to 0.14. The ability of supplying the load during the peaks of consumption proves as well the reliability of the computed combination of PV panels and batteries.

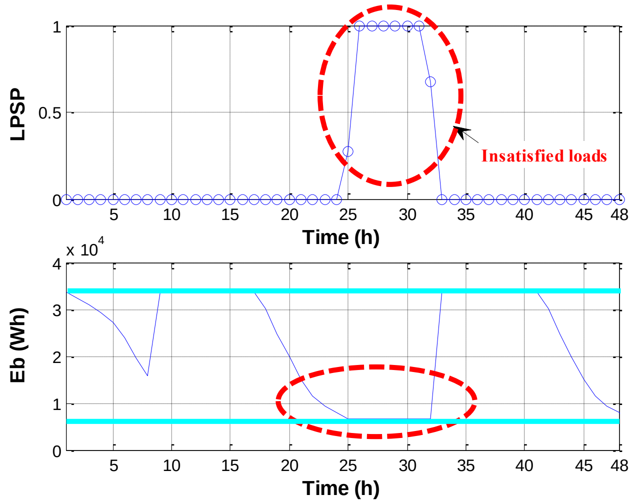


**Fig. 8.** LPSP and Energy stored in batteries Eb (Watt hour) of battery bank during two days ($N_{bat}$=28 and $N_{PV}$=24)

Usually in literature, the sizing of the battery depends on the number of days when the load is supplied only by batteries (usually 7 days for autonomous PV systems) [15]. This hypothesis can lead to an over sizing of batteries. As shown in the simulation results, the proposed methodology evaluates the batteries stored energy every hour of the simulation time. Therefore, the computed number of batteries guarantees the efficiency of the system.

The efficiency of the presented methodology is proven by the results of the rate of Loss of Power supply Probability evaluation during one and two days. By taking into account the random and unpredictable nature of the solar resource, the development of an optimal design of a PV stand-alone system for remote areas was presented in this paper.

## 5. Conclusion

The characteristics of the site for renewable energy production in remote area are very important. Its influence on the system design is remarkable. In this context, an optimal sizing of a PV stand-alone system with battery storage is investigated. The methodology proposed, in this paper, is based on statistical modeling of the solar radiation, same as the choice of the PV panel. The constraints related to the functioning of the battery have been totally respected in the simulation program to insure higher performances. The minimal percentage of LPSP proved the effectiveness of the proposed methodology. The number of PV panels and batteries required for a day can be sufficient even for two days. Supplying the loads during peaks of consumption has always been a problem for electricity production. The computed combination proved its efficiency in this point. These performances are due to the use of statistical techniques that allows taking into account the random nature of the resource. It can be said that the statistical techniques permit avoiding the over sizing of the system and as primary consequence avoiding a supplement cost. It could be interesting as perspective, the validation of the computed combination (PV panels and batteries numbers) in terms of cost.

## Acknowledgements

The authors thank the minister of high studies and scientific research and CMCU-12G1103 project for supporting this work.

## References

[1]. WWF, The Energy Report 100% Renewable Energy by 2050, 2011. (Report)
[2]. EPIA, Global Market Outlook For Photovoltaics until 2016, May 2012. (Report)
[3]. REN21( renewable Energy Policu Network for the 21st Century), Renewables 2012 Global Status Report, 2012. (Report)
[4]. Malek Belouda, Jamel Belhadj, Bruno Sareni, Xavier Roboam, "Battery sizing for a stand alone passive wind system using statistical techniques", International Multi-Conference on systems, Signals & Devices, 2011. (Conference Paper)
[5]. G. B. Shrestha, L. Goel, "A study on optimal sizing of stand-alone photovoltaic Stations". IEEE Transactions on Energy Conversion, Vol. 13, No. 4, pp. 373-378, December 1998. (Article)
[6]. Bogdan S. Borowy, Ziyad M. Salameh, "Optimum photovoltaic array size for a Wind/PV hybrid system ", IEEE Transactions on Energy Conversion, vol. 9, N. 3, 482-488, September 1994. (Article)
[7]. Bogdan S. Borowy, Ziyad M. Salameh, " Methodology for optimally sizing the combination of a battery bank and PV array in a Wind/PV hybrid system", IEEE Transactions on Energy Conversion, vol. 11, No. 2, pp. 367-375, June 1996. (Article)
[8]. Rachid Belfkira, Cristian Nichita, Pascal Reghem, Georges Barakat, "Modeling and optimal sizing of hybrid renewable energy system", 13th International Power Electronics and Motion Control Conference (EPE-PEMC 2008). (Conference Paper)
[9]. Hongxing Yang, Wei Zhou, Lin Lu, Zhaohong Fang, "Optimal sizing method for stand-alone hybrid solar–wind system with LPSP technology by using genetic algorithm", Solar Energy 82, 354–367, 2008. (Article)
[10]. Ramos Hernanz, Campayo Martín, Zamora Belver, Larrañaga Lesaka, Zulueta Guerrero, Puelles Pérez, "Modeling of Photovoltaic Module ", International Conference on Renewable Energies and Power Quality (ICREPQ'10), 23rd to 25th March, 2010. (Conference Paper)
[11]. Kinal Kachhiya, Makarand Lokhande, Mukesh Patel, "MATLAB/Simulink model of solar PV

module and MPPT algorithm", National Conference on Recent Trends in Engineering & Technology, 13-14 May 2011. (Conference Paper)

[12]. Sree Manju B, Ramaprabha R, Mathur B.L, " Design and Modeling of Standalone Solar Photovoltaic Charging System", International Journal of Computer applications(0975-8887), vol 18, No.2, March 2011. (Article)

[13]. Ziad M. Salameh, Bogdan S. Borowy, Atia R.A. Amin, "Photovoltaic Module-Site Matching Based on the capacity factor", IEEE Transactions on Energy Conversions, Vol.10, No. 2, June 1995. (Article)

[14]. http://www.investintunisia.tn/site/fr/article.php

[15]. D. Abbes, Contribution au dimensionnement et à l'optimisation des systèmes hybrides éoliens-photovoltaïques avec batteries pour l'habitat résidentiel autonome, National Engineering School of Poitiers, 2012. (Thesis)